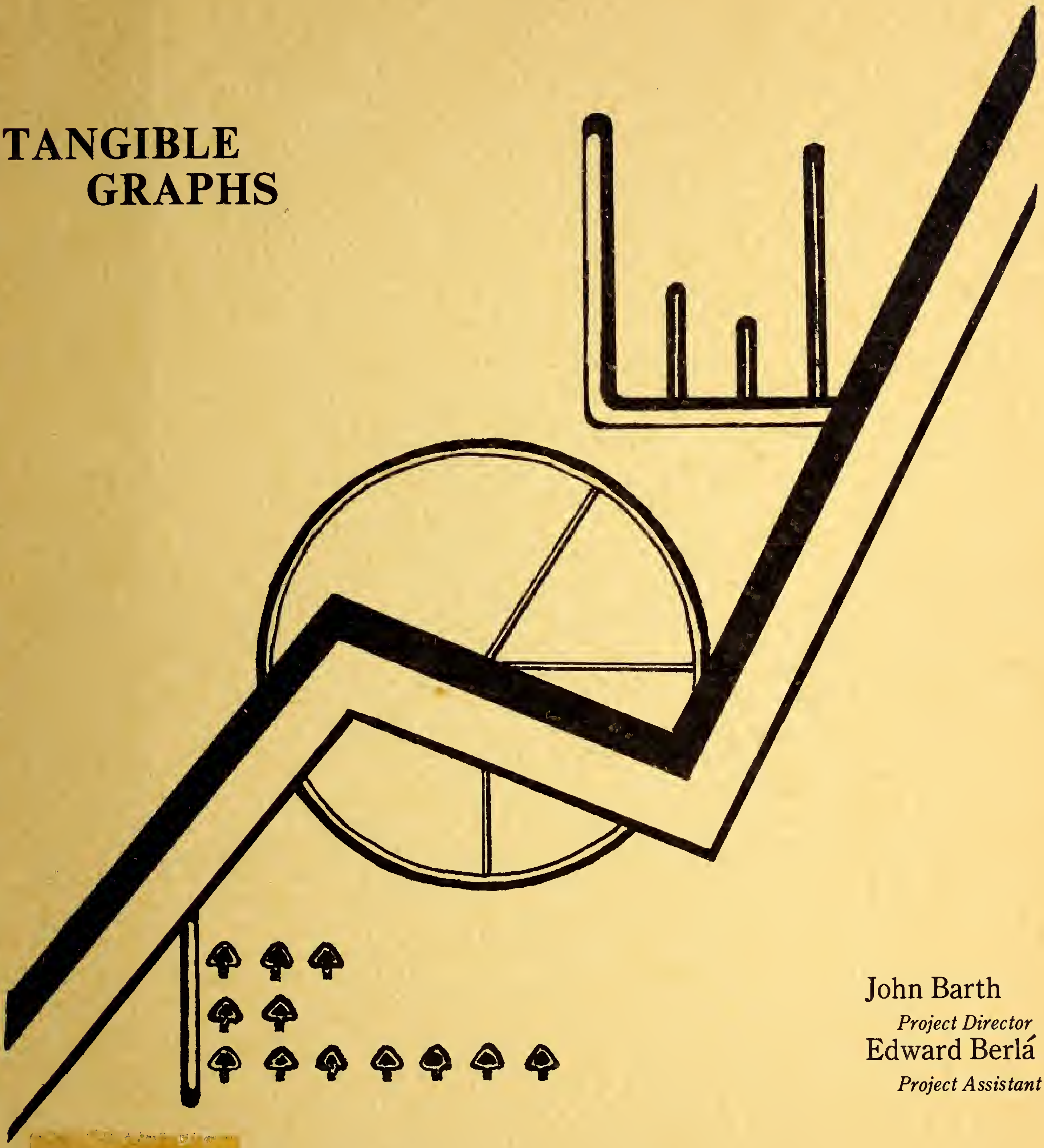
Teacher's Guide

TANGIBLE GRAPHS



John Barth
Project Director
Edward Berlá
Project Assistant

HV1705
.B282
B455
1982

American Printing House for the Blind
Louisville, Kentucky

HV1705
B282
B455
1982

Copy #1
12/89

TANGIBLE GRAPHS

John L. Barth, Ph.D.
Project Director

Edward P. Berla', Ph.D.
Project Assistant

AMERICAN FOUNDATION FOR THE BLIND
15 WEST 16th STREET
NEW YORK, NY 10011

American Printing House for the Blind
Louisville, Kentucky
1982

Key Concepts

Concept	Page
lines	3
shapes	9
patterns	10
key	12
below	22
above	22
on	23
through	23
inside	24
outside	24
left	25
right	25
between	25
horizontal	26
vertical	27
diagonal	28
longer	30
higher	30
shorter	30
lower	30
straight	36
curved	36
broken	38
intersect	39
perpendicular	40
number line	53
interval marker	54
pictograph	59
title	62
symbol	62
bar graph	66
horizontal bar graph	66
vertical bar graph	66
origin	67
horizontal axis	67
x-axis	67
vertical axis	67
y-axis	67
label	69
row	75
column	75
table	76
grid	84
values	85
coordinate values	86
line graph	93
trend	99
decreasing	99
increasing	99
circle graph	104

Concept	Page
pie graph	105
slope	127
negative slope	132
positive slope	132
range	132

Preface

Graphic displays in the form of graphs, maps, diagrams, and tables represent an indispensable means for communicating information. Their proliferation is expected to continue at an increasing rate. Quite simply, graphic materials have been recognized as having the capability of conveying certain types of information that cannot be easily or efficiently represented in verbal form. Any person, regardless of visual status, is placed at a disadvantage if he or she is not taught how to extract the wealth of information available in graphic displays.

Graphs represent a significant percentage of these displays. They appear frequently in textbooks and other educational media, in such diverse subject areas as mathematics, social studies, physical sciences, biological sciences, and business. There is a growing use of graphs and tables to summarize data from every branch of science, industry, business, and government. They are increasingly encountered in newspapers and magazines. The National Council of Supervisors of Mathematics considers the reading and interpretation of graphs as one of the basic skill areas that need to be emphasized in mathematics education. Graph literacy is thus expected of any educated person and is in fact tested in standardized educational measures such as the Stanford Achievement Test, the California Achievement Tests, the SAT, and the ACT.

The purpose of this program is to turn blind students into competent graph readers, through a carefully sequenced instructional approach. Fundamental graph reading skills and concepts are introduced in small steps, with each new step building upon previous ones. The student is thus brought along slowly from elementary skills such as tactual discrimination and line tracking

to more advanced skills such as the interpretation of bar graphs and multiple line graphs. The program is replete with tangible graphic displays. Approximately half of them are embossed in paper and half in plastic to give the student practice with both types of medium. Accessory materials that allow the student to construct his or her own graphs are also included in the instructional package. After completing the program, the student should be able to interpret all four main types of graph: pictograph, bar graph, circle graph, and line graph. It is also expected that the program will have some positive carry-over effects to the reading of other types of tangible displays, such as maps and diagrams.

This Teacher's Guide contains a complete print version of the braille program. Page numbers corresponding to the braille text are provided in the margins for easy cross reference. In addition, Teacher Notes have been inserted throughout the Guide. These notes contain important information and should be paid special attention.

TANGIBLE GRAPHS

John L. Barth, Ph.D.
Project Director

Edward P. Berla', Ph.D.
Project Assistant

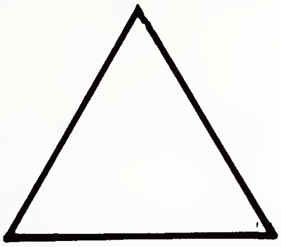
As you know, there are many ways that you can learn about things in the world. You are probably most familiar with braille and recorded tapes. By reading braille and listening to tapes, you can learn and enjoy all kinds of information. But braille and tapes are not the only means of presenting information so that you can learn new and interesting things. You can also learn from pictures that you feel with your fingertips. You probably have heard them called raised line drawings. Examine the following drawings: [p.2]

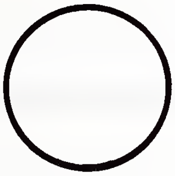
TEACHER NOTE

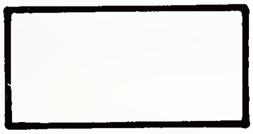
Make sure that the student understands the meaning of the word "following." It will be used throughout the program to refer to the next raised line drawing.

A Stokes Place Holder aid has been provided in the program package. The sheet of metal is very useful for providing a flat, rigid surface upon which to examine a page. Because

of the large number of raised drawings in the text, a page does not lie completely flat and thus may be difficult to read. By placing the metal sheet underneath the page being studied, this problem is eliminated. The magnetic marker included with the sheet facilitates place keeping. It is particularly useful in situations where the student must refer back and forth between a graphic and the relevant text.





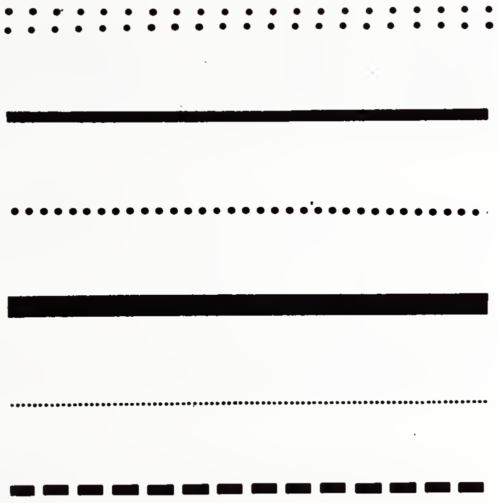


What do you feel? The first shape is a triangle. The second shape is a circle. The third shape is a rectangle. These are raised line drawings. They help you understand what a triangle, circle, and rectangle are like. It would be difficult to describe these ideas by only using words. A picture often makes things easier to learn, and more fun. But just as with braille, you have to learn how to read them. That is the purpose of this program, to teach you how to read a special kind of raised line [p.3] drawing, called a graph. Graphs appear everywhere in print. They're in magazines, newspapers, and school books. All school children are taught how to read them. By the end of this program, you'll be able to read them, too.

TEACHER NOTE

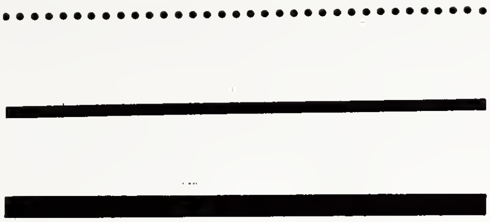
This double solid line will be used throughout the program to indicate a change in subject matter. It is advisable to end an instructional period at one of these transition points.

Let's begin by examining the kinds of things that make up a raised line drawing. There are six lines below. Carefully feel each one.



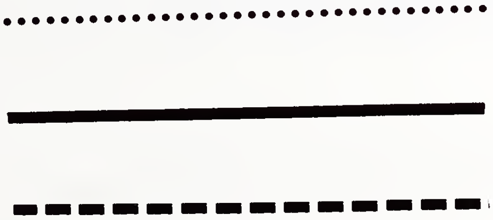
What do you notice about these lines? Do they all feel different? Why do they feel different? [p.4]

Let's explore lines more closely. Feel the following three lines:



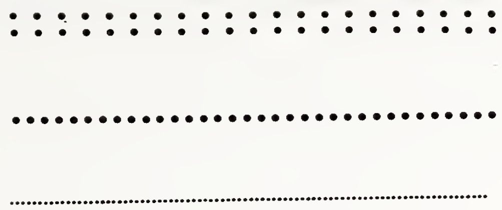
The first line feels rough. It is made up of many small dots. The second line is different. It feels smooth. Now feel the second and third lines and compare them. Both feel smooth. Are they the same? No. The third line is wider or thicker than the second line.

Examine these three lines: [p.5]



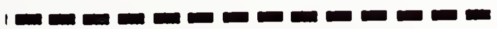
Compare the first and second lines with the third line. The third line is not as rough as the first line, but it is also not as smooth as the second line. This type of line is called a dashed line.

Here are three more lines:

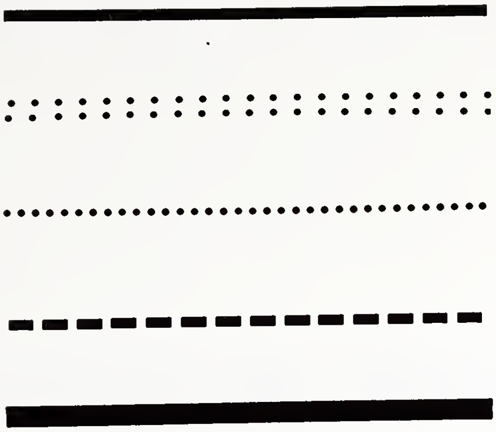


Feel the first line. It feels rough, doesn't it? Is it the same as the second line? No. If you examine it closely, you will see that it is made up of two lines of dots placed next to each other. This makes it feel like a wide rough line. Now [p.6] let's make one more comparison. Feel the third line. In what way is it different from the second line? Both of the lines are rough, made up of dots. But the second line stands out more, doesn't it? It is easier to feel. The reason for this is that it is raised higher than the third line.

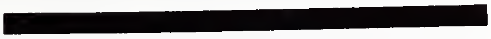
So, you see, many different kinds of lines can be made for you to feel. Before we go on, let's practice examining lines by playing a matching game. Feel this line:



Now see if you can find a line just like it among the following five lines: [p.7]



Now feel this line:



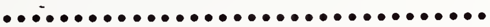
Can you find the line just like it among the preceding five lines?

TEACHER NOTE

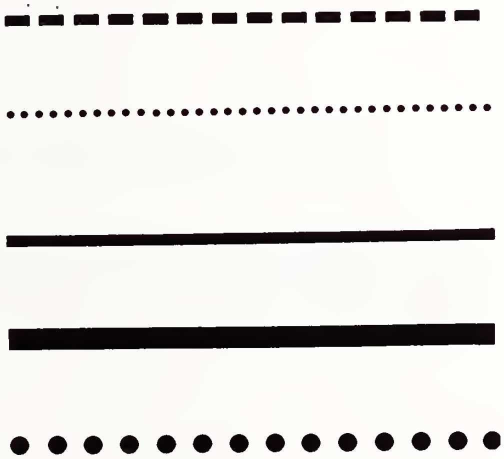
Make sure the student understands the meaning of the word "preceding." It will be used throughout the program to refer to the raised line drawing last encountered.

Let's play the matching game of lines one more time. Notice that this page is made of paper. Lines can be made in paper as well as in plastic. [p.8]

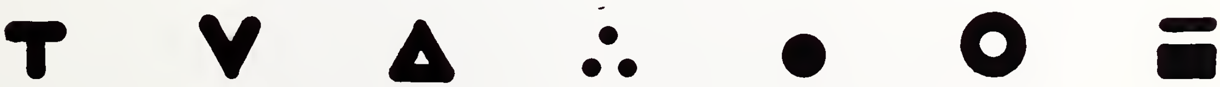
Now examine this line:



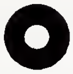
Find the line that feels just like it among the following five lines:



Lines are not the only things used in a raised line drawing. Shapes that you can feel with your fingertips are also used. Examine the following shapes. Think about why they feel different from each other. [p.9]

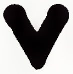


Let's see how well you can tell them apart. Examine this shape:



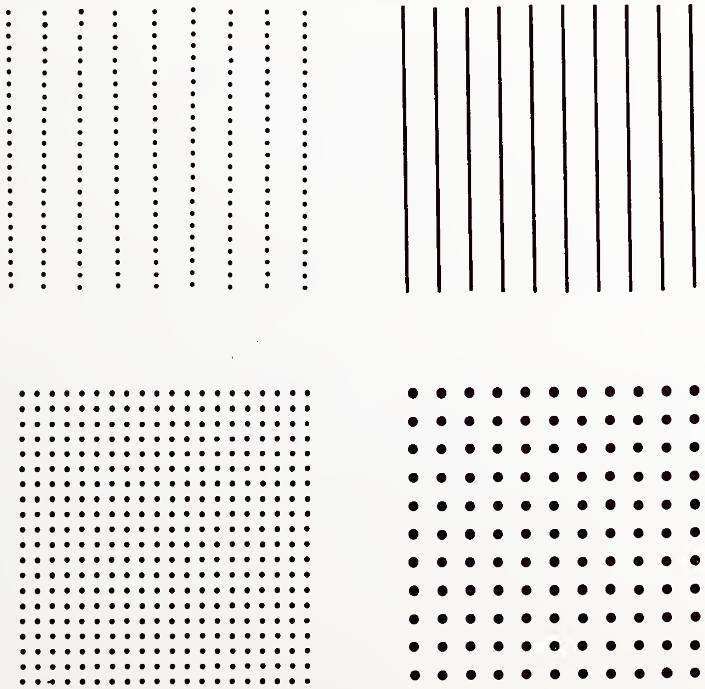
Now find the shape that is just like it among the group of shapes you examined earlier.

Examine this shape:



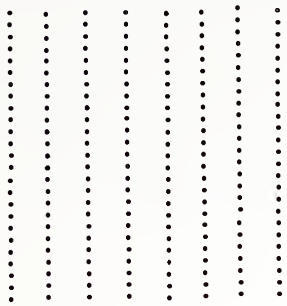
Which of the shapes feels exactly like this shape?

Patterns are also often found in raised line drawings. Carefully examine each of the following patterns: [p.10]



The first thing you should notice is that these patterns are not raised as high as the lines and shapes you examined earlier. This is done so that you can more easily tell them apart from the lines and shapes.

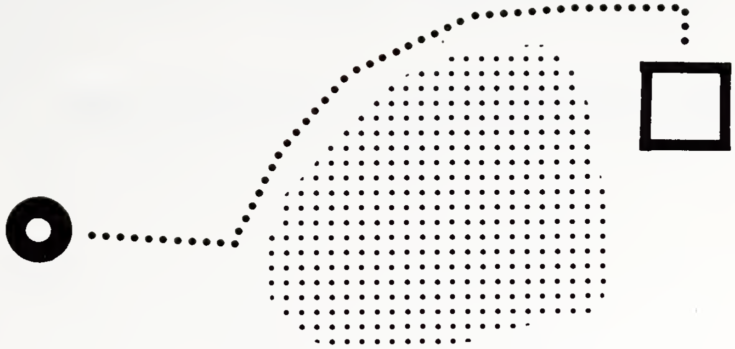
Now feel this pattern: [p.11]



Which of the previous four patterns is the same as this one?

Raised line drawings, then, can be made up of lines, shapes, and patterns.

Examine the following drawing. It is made up of two shapes, one line, and one pattern. See if you can locate them in the drawing.



Key:

◉ house

•••••••••••• road

□ school

▨ playground

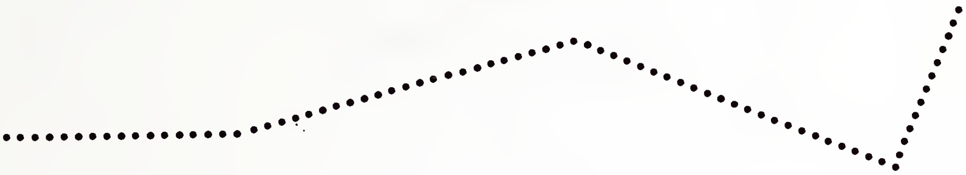
By itself the drawing doesn't tell you much. This is because you don't know what the shapes, line, and pattern mean. To make sense out of a raised line drawing, you have to know what each of them represents. This is the purpose of a key. A key tells you what the lines, shapes, and patterns in a drawing mean. Look at the key located below the drawing. The key tells you that one shape, the circle, is used to represent a house. The other shape, the square, represents a school building. The dotted line represents a road. What does the pattern represent? [p.13]

According to the key, the pattern is used in the drawing to show a playground. Look at the drawing again. Now you should recognize that it is a map. It shows that you can walk from the house to the school building by following the road around the playground. This is one example of how lines, shapes, and patterns are used in a drawing to tell you something.

In the drawing you just examined, you might want to follow the line from the house to the school building. Being able to follow a line accurately with your fingertips is very important. If you lose track of a line when you're following it, you'll quickly become confused. The best way to follow a line is by using two fingers.

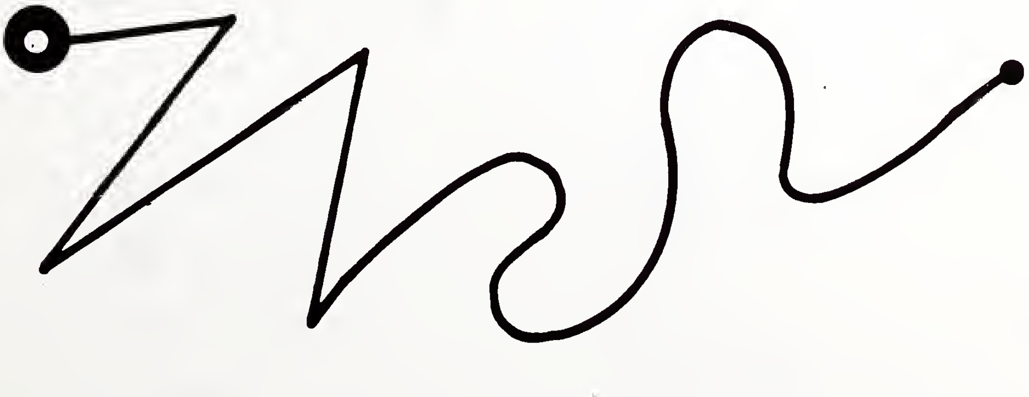
Put your right index finger on the beginning of the following line. Your index finger is the one closest to your thumb.

[p.14]

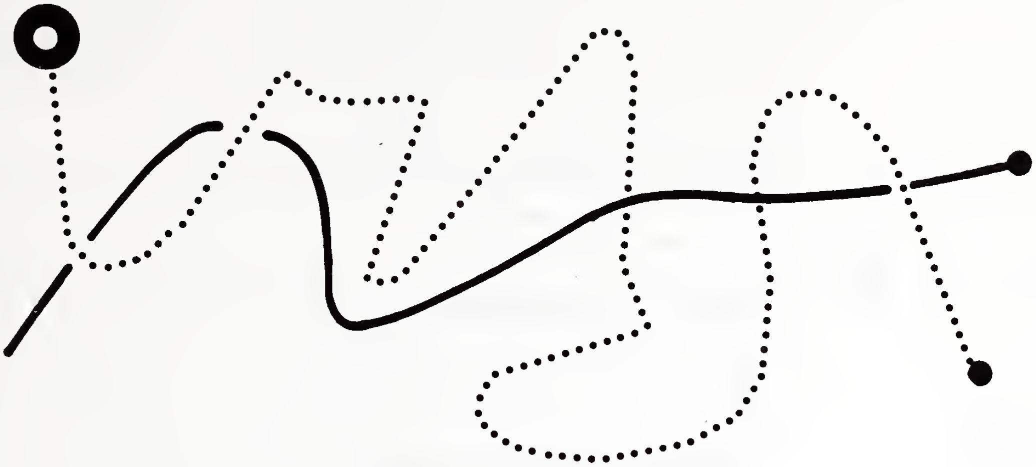


Now put the index finger of your left hand next to your right finger. Move both of your fingers along the line. Your right index finger will lead the way and your left index finger will trail behind. This will make it easier for you to keep track of the line. If your right finger loses its way, your left finger will still be on the line. When you reach the end of the line, go back to the beginning and trace the line again, using both fingers. Try to increase your speed without losing track of the line.

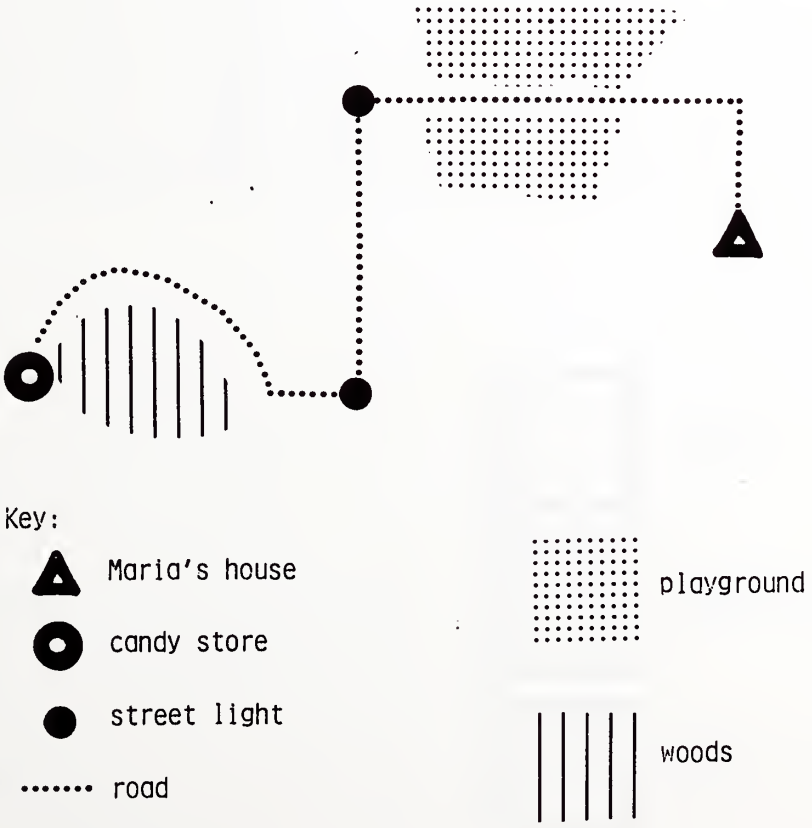
Here is another line. Start at the shape that feels like a circle and trace the line with both fingers. [p.15]

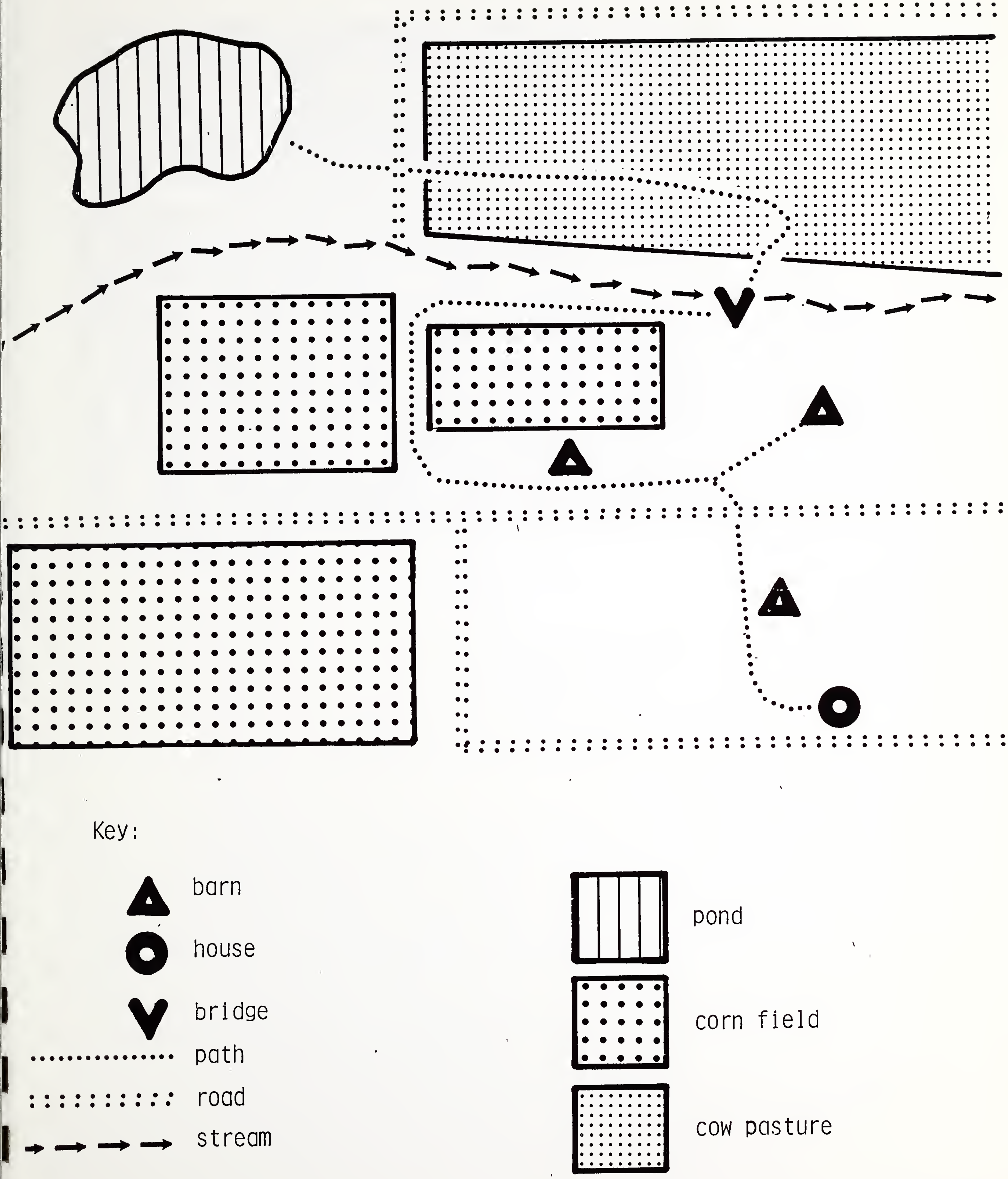


Trace the following dotted line. It starts at the circle and ends at the dot. Don't be confused by the solid line.



Here is a map of Maria's neighborhood. See if you can find your way from Maria's house to the candy store. Don't forget to study the key first. [p.16]

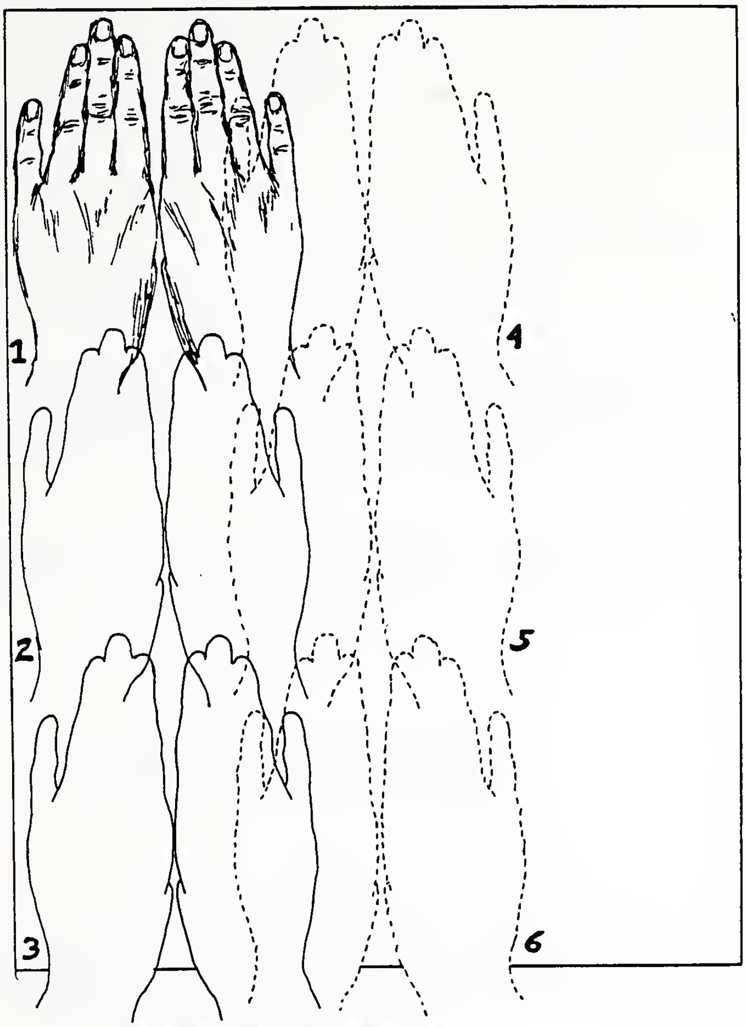

Key:
Maria's house
candy store
street light
road
playground
woods


Key:
barn
house
bridge
path
road
stream
pond
corn field
cow pasture

Now let's look at a drawing of another map. This time the line [p.18] you are going to follow will pass by shapes, go between patterns, and cross other lines. There is a drawing on the page to your left. This is a map of Mr. Green's farm. On the farm there are cornfields, a cow pasture, a pond, a house, barns, a stream, and even a bridge. There are also several roads and a path. Examine the key at the bottom of the page and find out what each of the shapes, lines, and patterns represents on Mr. Green's farm.

Now examine the map above the key. The first thing you should do is scan the entire map with both hands to find out how much of the page it takes up. In other words, find the top, bottom, left, and right borders of the map. After you have done that, examine the parts of it more closely and get an idea of where the shapes, lines, and patterns are located in the drawing. To do this, place both of your hands on the upper left-hand corner of the drawing. Now slowly move your hands toward you and try to remember what you feel. After you have reached the bottom of the map, move your hands back up to the top and place them in the center of the page. Now move them towards you again. After [p.19] you reach the bottom of the map, go back to the top one more time and place your hands on the upper right-hand corner of the drawing. Move them towards you. Examining a drawing in this way will help you get an idea of where all of the parts are located. It is always a good way to start. For practice, go back and try it one more time.

TEACHER NOTE

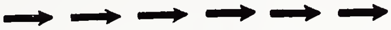


Systematic examination of a tactile display.

Now let's see how well you can use this map of Mr. Green's farm. Your task will be to follow the path from the pond to Mr. Green's house without getting lost. So look at the key again and find out how the pond, the path, and Mr. Green's house are represented on the map. When you're ready, find the pond on the map and follow the path from the pond until you reach Mr. Green's house. Use both of your index fingers to follow the line.

Did you find your way to the house? If you did, you should have ended up at the shape that feels like a circle. Try it once more and try to go faster this time.

Before we go on, let's examine a special kind of line. Feel the following line. Move your fingers over it in both directions. [p.20]

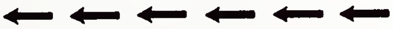


When you move your fingers to the right, does it feel different then when you move your fingers to the left?

Try it again. You should notice that the line feels rougher in one direction and smoother in the other direction. Which way feels smoother? If you said "to the right" you are correct. This is a line that tells you which direction to go, just like an arrow.

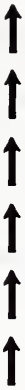
Here is a line with an arrow on one end. The arrow is pointing to the right. So it also tells you that the line is going from left to right.

Now examine the following line. Imagine that this line is a stream of water coming out of a hose. Which way is the water moving: to the right, or to the left? [p.21]



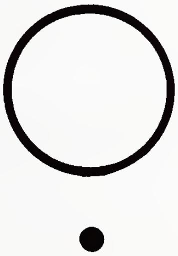
This is how it would look if we used an arrow:

Feel this line. Which way is the water moving now: up or down?

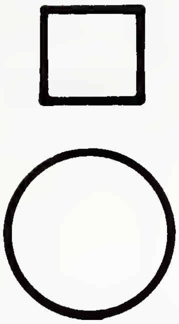


It feels smoother going up, so that is the direction the water is going. [p.22]

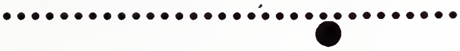
Examine the following dot and circle. The dot is below the circle.



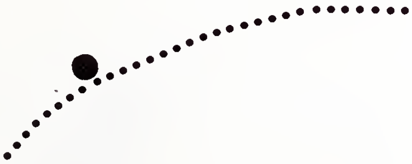
Now examine the following circle and square. The square is above the circle.



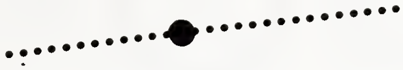
Here is a line and a dot. Is the dot above or below the line? [p.23]



Is the dot above or below this line?

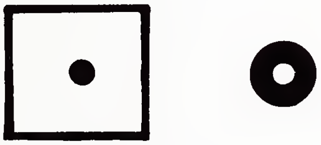


Here is another dot and line. Where is the dot?



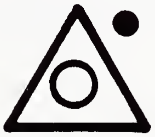
This dot is neither above nor below the line. It is on the line. In other words, the line passes through the dot.

Examine the following shapes: [p.24]

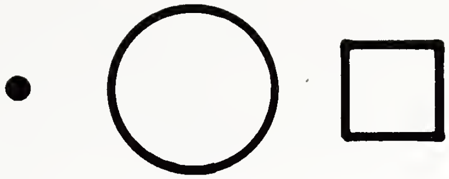


The dot is inside the square. The circle is outside the square.

Look at these shapes. Which shape is outside the triangle?

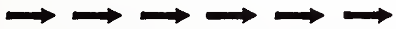


Examine the following dot, circle, and square. [p.25]

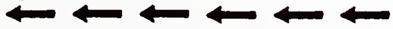


The dot is to the left of the circle. The square is to the right of the circle. The circle is between the square and the dot.

Trace this line:



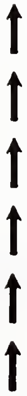
This is our special line. It feels smoother in one direction. The line you just examined goes from left to right across the page. Feel this line: [p.26]



This line goes from right to left across the page. Any line that goes from left to right or right to left without going up or down is called a horizontal line.

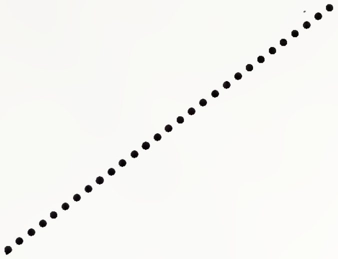
Trace the following line. Notice that it goes from the top of the page toward the bottom of the page.

Now trace this line. [p.27]

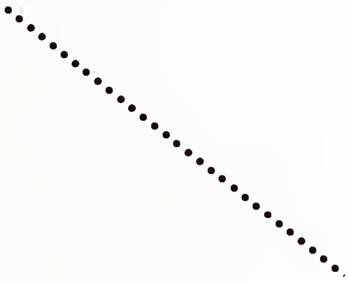


This line goes from the bottom of the page to the top of the page. Any line that goes straight up or straight down without going to the right or left is called a vertical line.

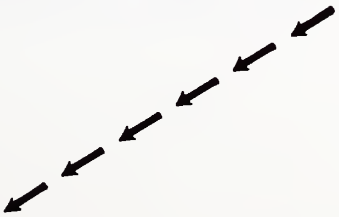
Trace the following line. Notice that it is neither a horizontal line nor a vertical line. [p.28]



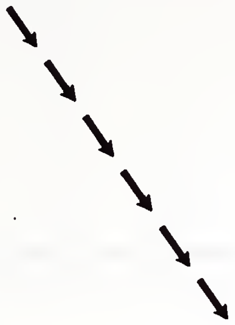
Here is another line that is neither a horizontal line nor a vertical line.



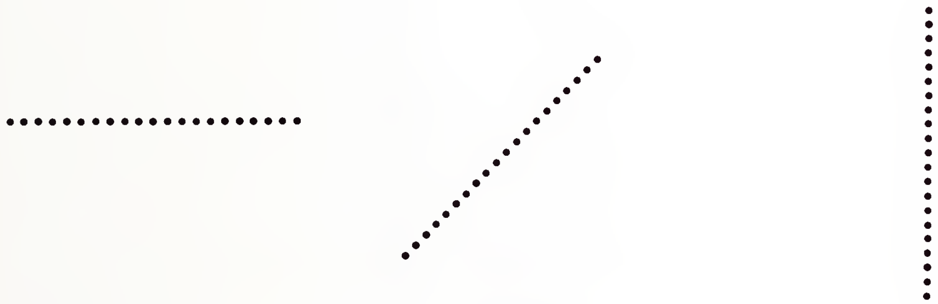
Trace the following line and notice that it goes from the upper right hand corner down toward the lower left hand corner. This line is called a diagonal line. [p.29]



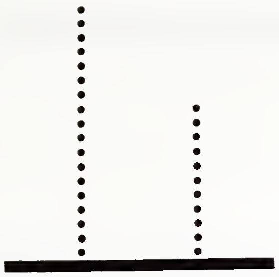
Another kind of diagonal line is one that starts at the upper left hand corner and points toward the lower right hand corner. Trace this diagonal line:



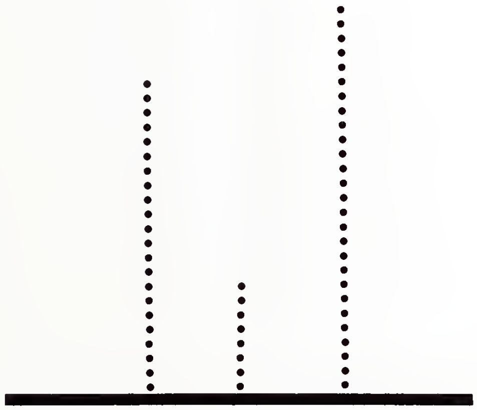
Examine the following three lines. Which one is the vertical, which one is the horizontal, and which one is the diagonal? [p.30]

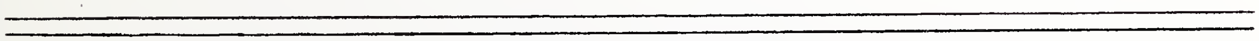


Feel the two lines shown below. The first line is longer than the second line. It is also higher than the second line. The second line is shorter than the first line. It is also lower than the first line.

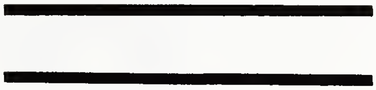


Examine the following three lines. Which is the highest and which is the lowest? [p.31]





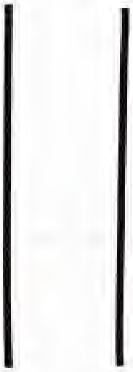
Here are two horizontal lines. Trace these two lines and notice how far apart they are.



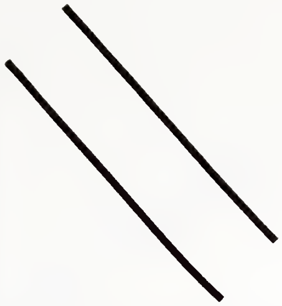
Are the two lines the same distance apart on the left side as they are on the right side?

When two straight lines are the same distance apart at both ends, they are said to be parallel. [p.32]

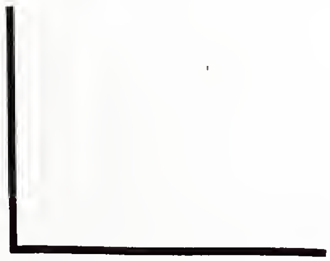
Trace the following two vertical lines. Are they the same distance apart at both ends? Are they parallel?



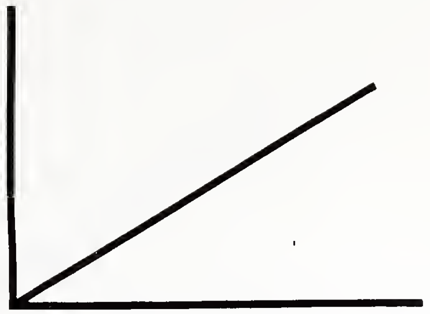
Here are two diagonal lines. Are they parallel? [p.33]



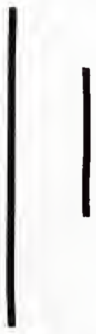
Be careful now. Here are two more lines. Trace them with your fingers. Are they parallel?



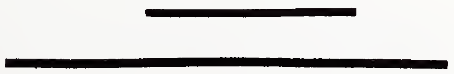
Here are three lines that all touch each other. Pick out the vertical, the horizontal and the diagonal. [p.34]



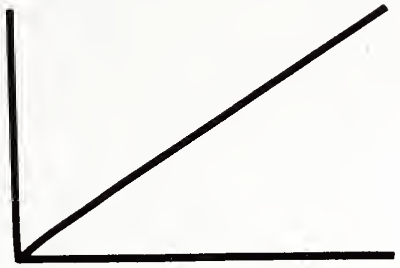
Here are two parallel, vertical lines. Which vertical line is longer?



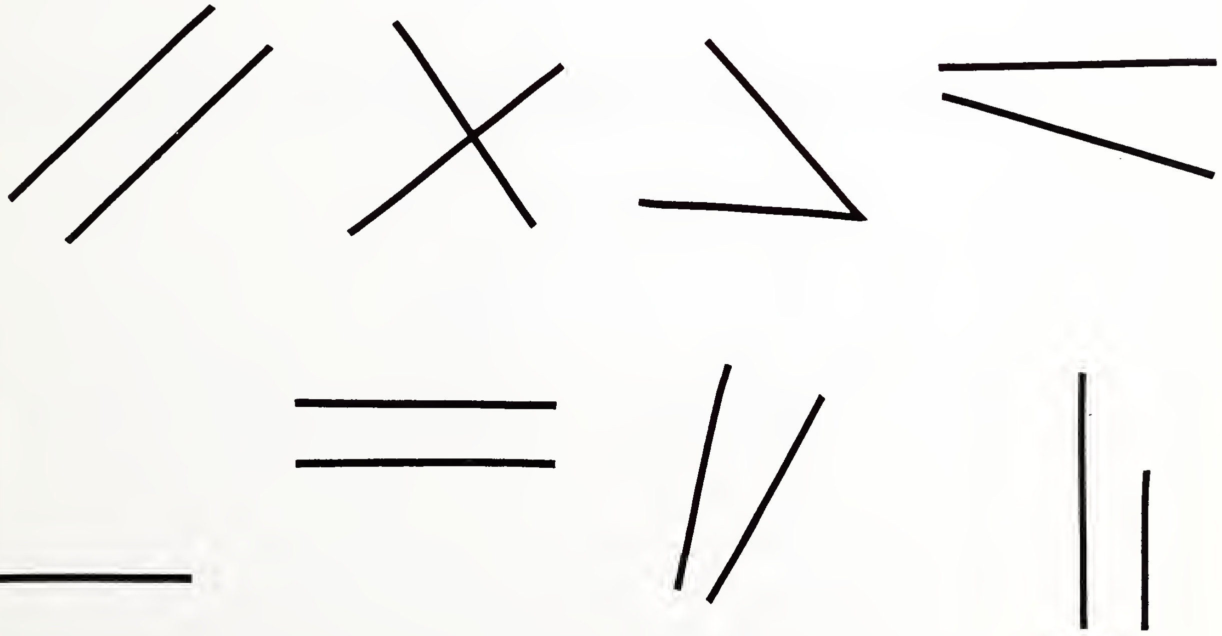
Here are two parallel, horizontal lines. Which horizontal line is shorter? [p.35]



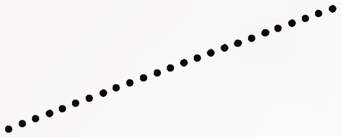
Here is a horizontal line, a vertical line, and a diagonal line. Which is the longest?

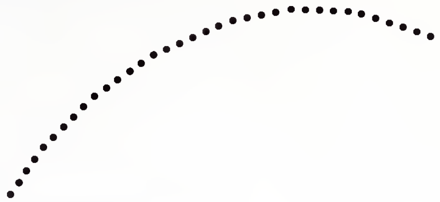


Examine these eight pairs of lines. Determine whether each pair of lines is parallel or not. When you find a pair of parallel lines, determine whether they are vertical, horizontal, or diagonal. [p.36]

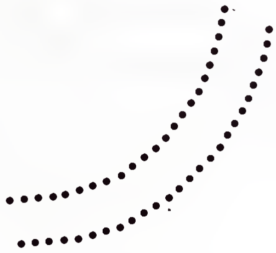


All of the preceding lines were straight. Lines can also be curved. Examine the following lines. The line on the left is straight. The line on the right is curved. [p.37]

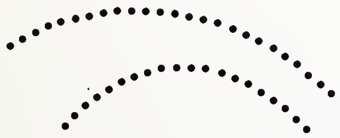


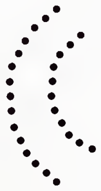


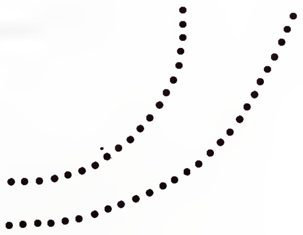
Curved lines can also be parallel to each other. Here are two curved lines that are parallel:

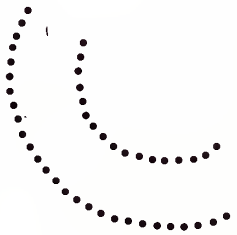


Determine which of the following pairs of curved lines are parallel.

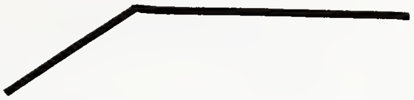




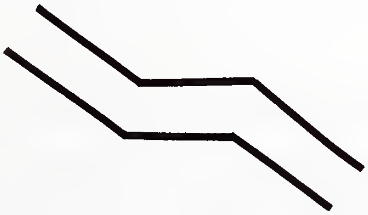




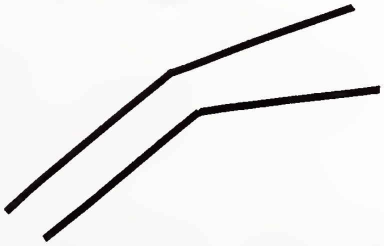
Here is another kind of line, called a broken line: [p.38]



Broken lines can also be parallel to each other. Here are two broken lines that are parallel:

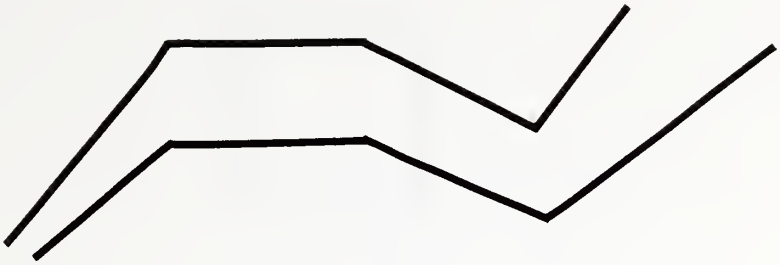


Examine this pair of broken lines. Are they parallel?

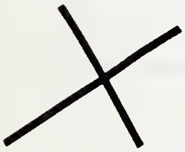


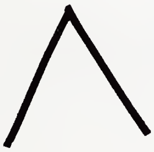
The first parts of the lines are parallel. The second parts are not. [p.39]

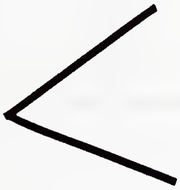
Here are two more broken lines. Which parts of the lines are parallel?

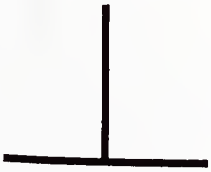


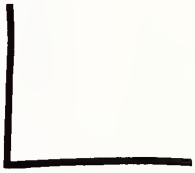
When two lines meet each other at a point they are said to intersect. All of the following pairs of lines intersect. Feel them carefully and find the point or place where they intersect. [p.40]



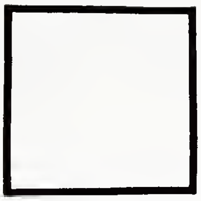






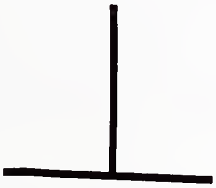


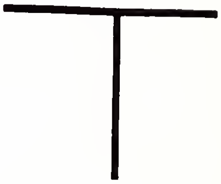
When a horizontal line intersects a vertical line, the lines are said to be perpendicular. Examine this square:



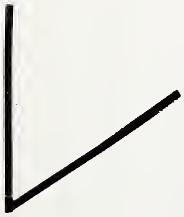
The corners of this square are formed by vertical and horizontal lines that intersect. So each corner is made up of perpendicular lines.

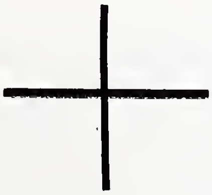
All of the following pairs of lines are perpendicular lines. [p.41]

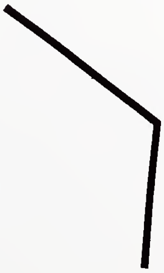


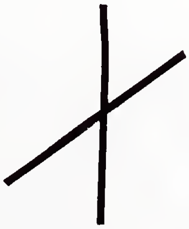


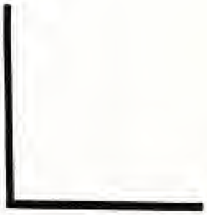
Which of these pairs of lines are perpendicular?











Can you think of some print capital letters that are made from perpendicular lines? Below are some print capital letters made from intersecting lines that are perpendicular. Can you name them? [p.42]

TEACHER NOTE

If the student is unfamiliar with print letters, you may want to skip this exercise.

L T H F E I

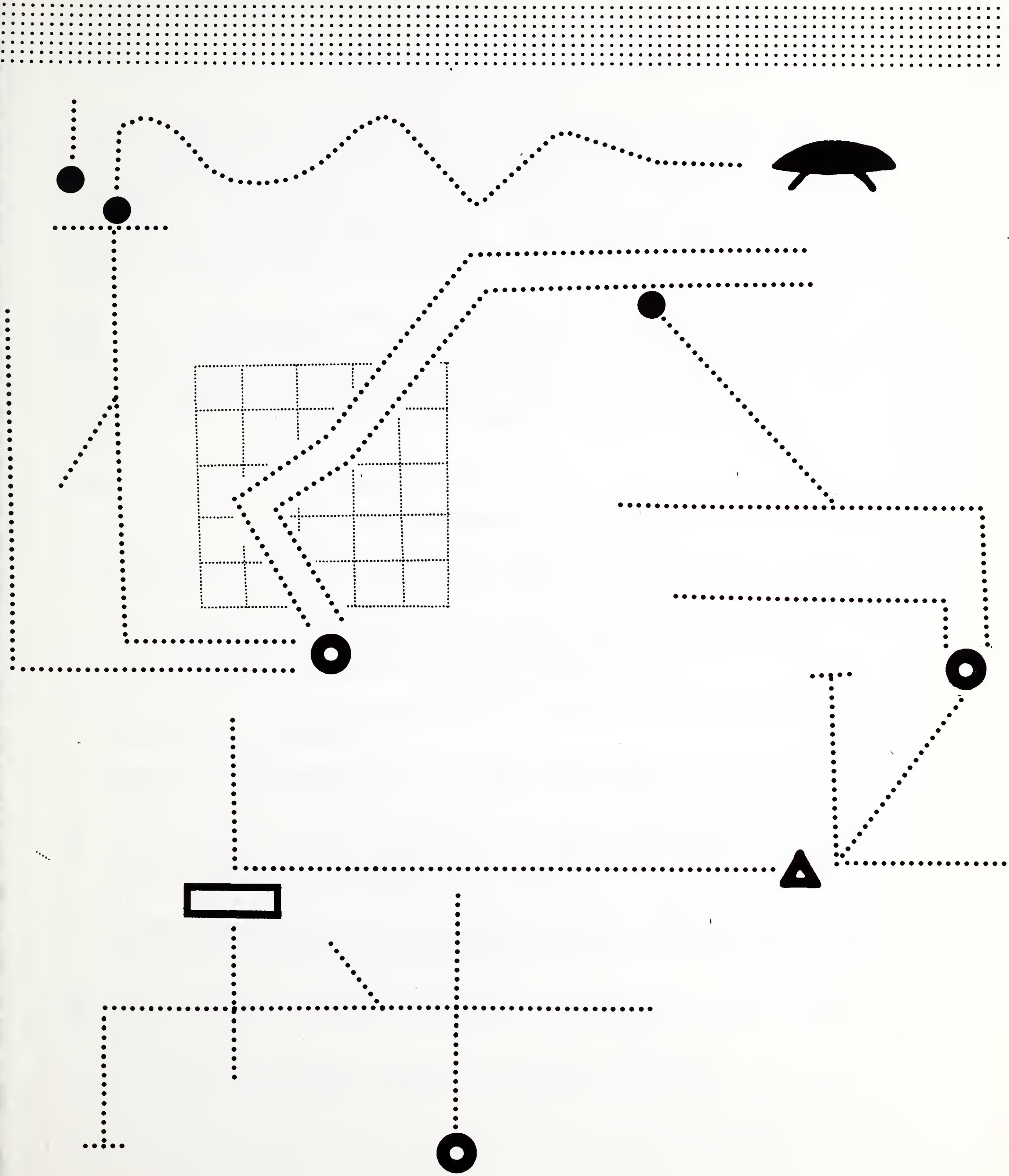
TEACHER NOTE

Participation Exercise: Using the cork board, pins, rubber bands, and graph paper, have the student construct vertical lines, horizontal lines, diagonal lines, parallel lines, and perpendicular lines (by lining up pins and/or by stretching rubber bands between pins).

Have the student locate and examine such features in the environment. For example, the corner of a table (perpendicular), the sloping back of a chair (diagonal).

TEACHER NOTE

The next two exercises serve several purposes. First, they provide additional exposure to and practice with the concepts presented in the preceding sections. Second, they provide practice in line tracking. Third, they give students practice in the following of directions and in the skill of referring back and forth between a drawing and the related text. To facilitate this latter skill, you might want to use the Stokes Place Holder aid provided with the program. In this case, the marker would be used to keep one's current position on the map while referring back to the directions.



If time does not permit, you can read the directions to the student as he or she uses the map.

The Meeting Place [p.44]

It was a long trip from his home planet, Zimka. So when the weary alien finally made it to Earth, he was very happy. Talking to somebody, even an Earthling, would be very nice after spending so many years alone in his spaceship. But he had to be cautious. If he landed his spaceship on the lawn of the White House, the Earthlings might panic. So he landed out in the country. Then he sent instructions and a map to the President, which explained how to find him.

See if you can follow the instructions, while reading the map, and find the spaceship. The top edge of the map has a rough texture on it. The bottom edge of the map is smooth. Make sure the bottom edge of the map is parallel with the edge of your desk or table. Now take a minute to explore the map to find out how big it is and what kinds of shapes, lines, and patterns are used.

Here are the instructions the alien sent: [p.45]

1. Start at the circle at the bottom of the map.
2. Above the circle is a vertical line. Follow it up the page until it intersects with a horizontal line.
3. Turn left at the intersection, and follow the horizontal line until you come to an intersection where there is a line that is perpendicular to the horizontal line.
4. Turn right at the intersection and follow the line until you reach the rectangle.
5. Above the rectangle is a vertical line and a horizontal line. Follow the horizontal line until you reach the triangle.
6. To the right of the triangle are three intersecting lines. One of them is a diagonal line. Find the diagonal line and follow it until you come to a circle.
7. Above the circle are two vertical lines. One is longer than the other. Follow the longer line until it intersects with a horizontal line and turn left. [p.46]
8. Follow the horizontal line until it intersects with a diagonal line.
9. Follow the diagonal line up the page to the dot.

10. Just above the dot are two parallel lines. Follow them to the left until they reach the pattern made up of little squares.

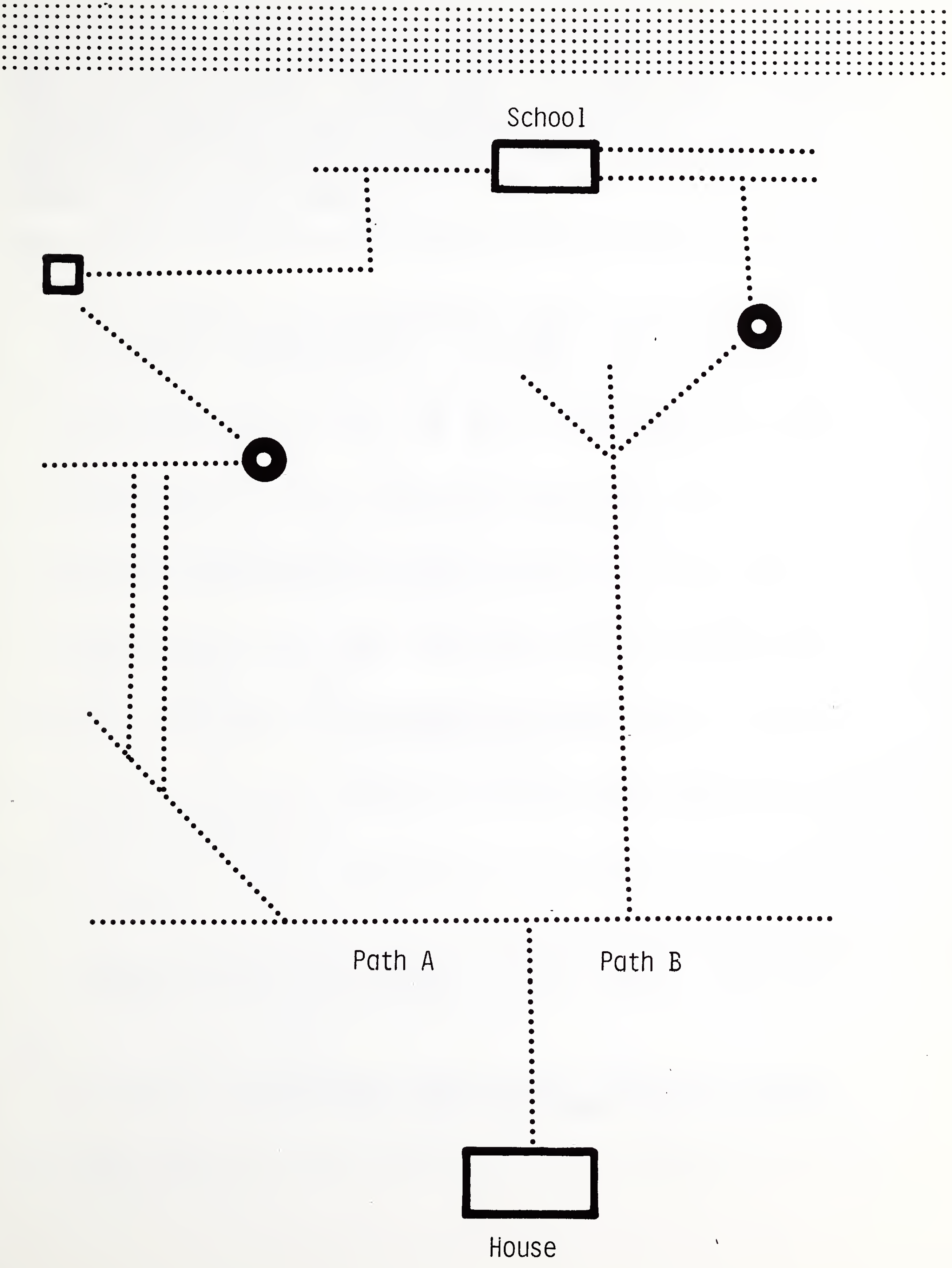
11. Follow the parallel lines as they wind their way through the pattern. Stop at the circle.

12. To the left of the circle are two parallel, horizontal lines. Follow the shorter one until you reach the vertical line.

13. Follow the vertical line up the page until you reach a line that is perpendicular to it.

14. Above this line are two dots. One dot is higher than the other dot. Find the one that is lower.

15. Follow the line above this dot. It leads you directly to the spaceship.


School
Path A
Path B
House

Which Path is Shorter

[p.48]

Erin moved to a new neighborhood and is enrolled in a new school. There are two paths that she can take to get to the new school. See if you can follow the paths and decide which is the shorter path for Erin to take. The top part of the map has a rough texture. Find the top part of the map. The bottom part of the map is smooth. Make sure the bottom edge of the map is parallel with the edge of your desk or table. Now take a minute to explore the map to find out how big it is and what kinds of shapes and lines are used.

Here are the directions for following the two paths to Erin's new school:

1. Erin's house is at the bottom of the map. The word house is spelled out at the bottom of the page. Above the word house is a small rectangle which is a symbol for Erin's house.
2. A vertical line extends from Erin's house and intersects with a horizontal line. One path, Path A, goes to the left and the second path, Path B, goes to the right. [p.49]
3. Follow path A first. Follow the horizontal line that goes to the left until you reach an intersection.
4. Follow the diagonal line until you reach the intersection with the two parallel vertical lines.
5. Follow the parallel vertical lines. When you come to an intersection, turn right, and follow that line until you reach a small circle.
6. Above the circle is a diagonal line that goes to the left. Follow it until you reach the square.
7. To the right of the square is a horizontal line. Follow it until you reach the vertical line which is perpendicular to the horizontal line.
8. Follow the vertical line until it intersects with a horizontal line.
9. Follow the horizontal line to the right until you reach the rectangle which is the symbol for the school. Above the rectangle is the word school. [p.50]

Now let's follow Path B.

1. Go back to the bottom of the page and find Erin's house. Follow the vertical line until you reach the intersection.
2. Turn right and follow the path until it intersects with the vertical line.

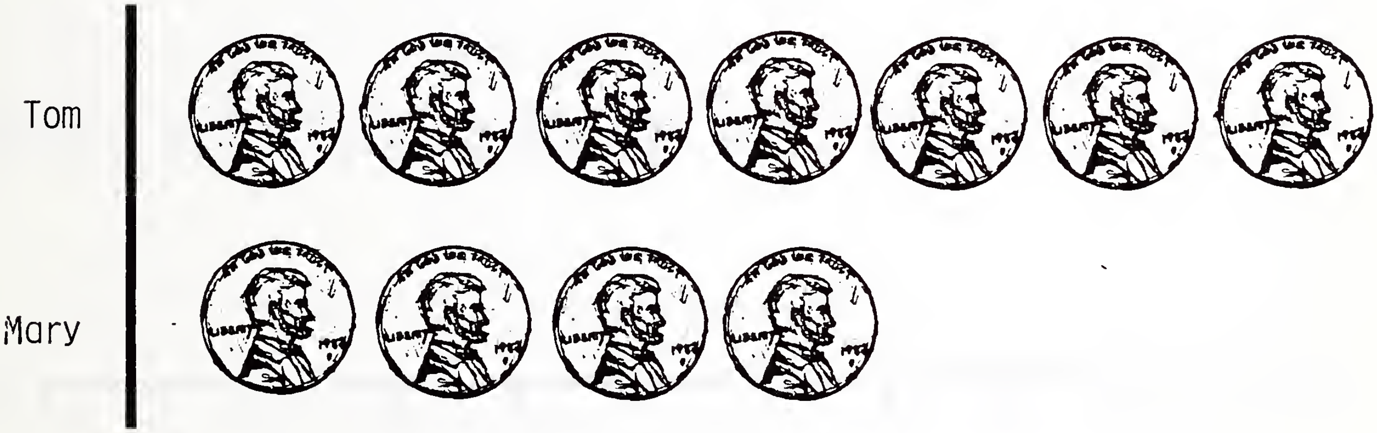
3. Follow the vertical line until you come to an intersection.

4. There is a diagonal line to the left, a vertical line, and a diagonal line to the right. Follow the right diagonal line until you reach the circle.

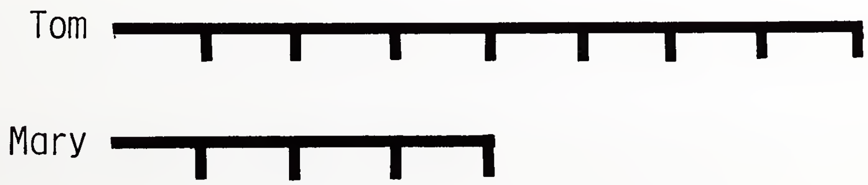
5. Above the circle is a vertical line. Follow the vertical line until you reach the intersection with the two horizontal parallel lines.

6. Follow the parallel lines to the left until you reach the rectangle which is the school. Can you decide which path is shorter? If not, follow them again. Which path should Erin take to school?

Tom and Mary took the pennies out of their pockets and lined them up on the table. Who had the most money? [p.51] Volume 2

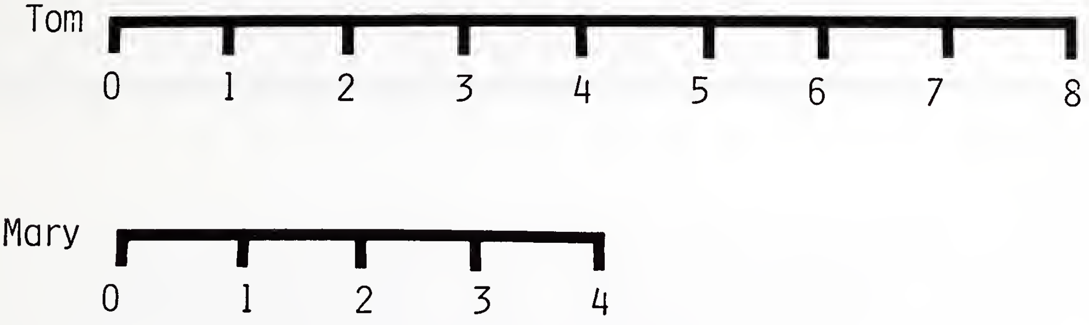


There is another way to do this. Tom and Mary each drew a straight line on a piece of paper and then put a mark on the line for each penny, like this: [p.52]

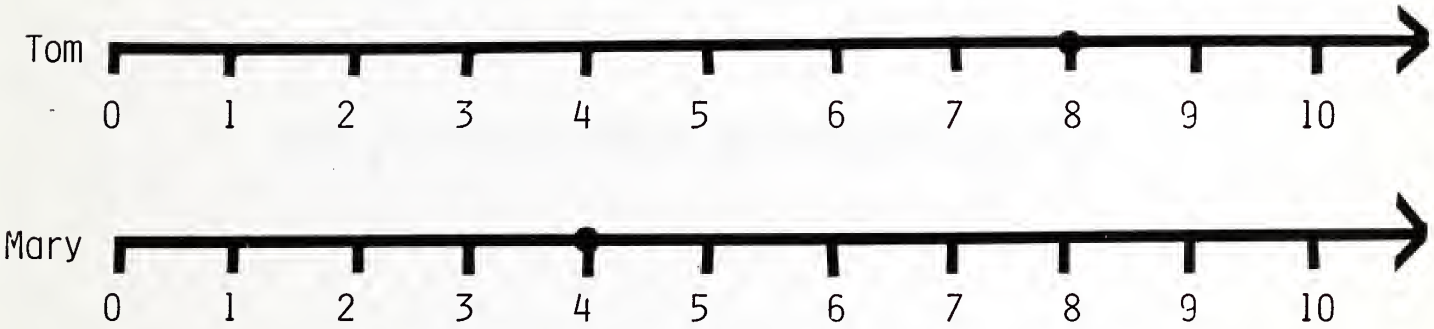


By counting the marks on the lines, you can tell how many pennies Tom and Mary had.

If there was a number below each mark, it would be even easier to tell.

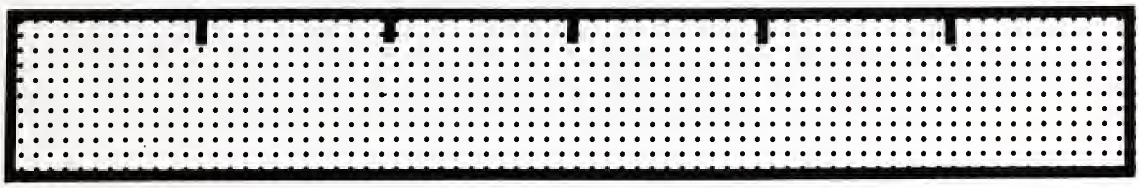

Tom
0 1 2 3 4 5 6 7 8
Mary
0 1 2 3 4

Here is another way to show the same thing. In this case, a dot is placed on the line above the number of pennies each had. [p.53]


Tom
0 1 2 3 4 5 6 7 8 9 10
Mary
0 1 2 3 4 5 6 7 8 9 10

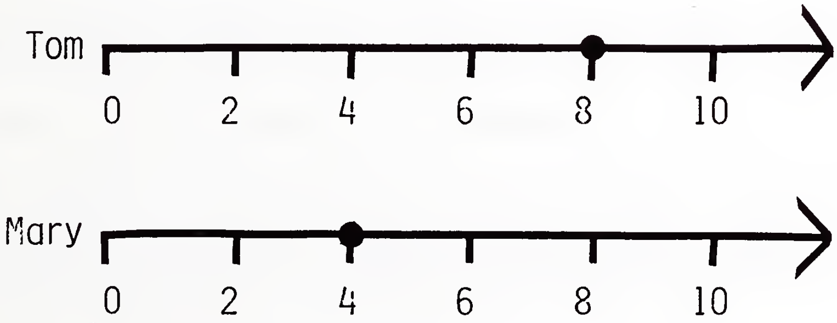
These lines are called number lines.

A ruler is a good example of a number line. On a ruler, each mark represents one inch. How long is the following ruler?



Put your finger at the beginning of the ruler. Now move your finger to the right until you feel the first marker. That marker tells you that you have moved your finger an interval of one inch. That is why it is called an interval marker. On a ruler each interval is one inch long. [p.54]

An interval on a number line does not always represent one unit. Suppose Tom and Mary made their number lines to look like this:



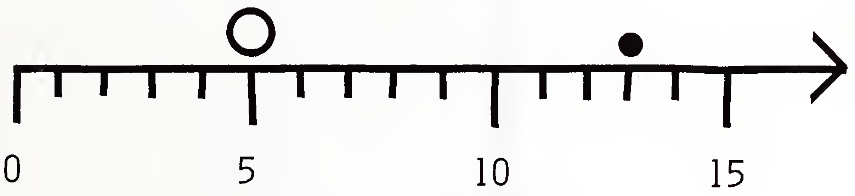
How many pennies are represented by each marker?

In this case, each marker and therefore each interval represents two pennies. In other words, each interval represents two units.

How many pennies are represented by two intervals?

How many intervals would be needed on the number line to represent 20 pennies? [p.55]

Ed is a sixth grade student who collects newspapers on the weekend for the school paper drive. He wanted to see how well he did on Saturday as compared to Sunday. So, hew drew a number line to show the number of papers collected. On this number line he drew a circle to show the number of papers collected on Sunday. He drew a dot to show the number of papers collected on Saturday. On which day did Ed collect more papers?



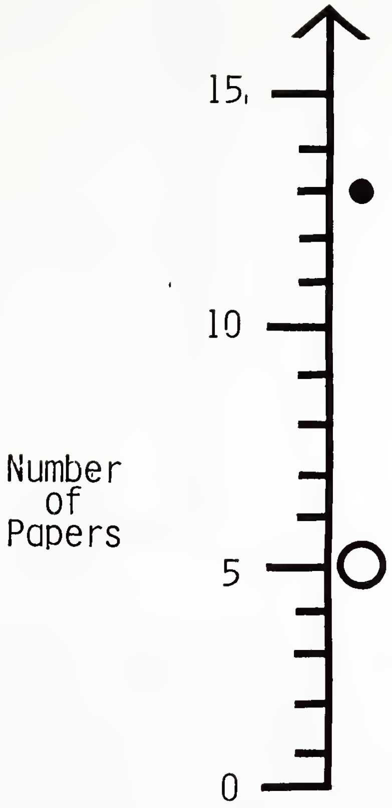
Number of Papers

Notice that not every interval marker has a number below it. It is not necessary to number every marker on a number line. [p.56]

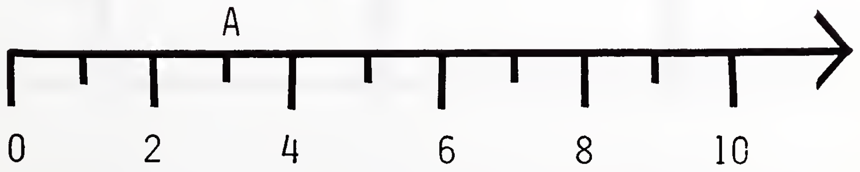
How many papers did Ed collect on Saturday?

How many more papers did he collect on Saturday than Sunday?

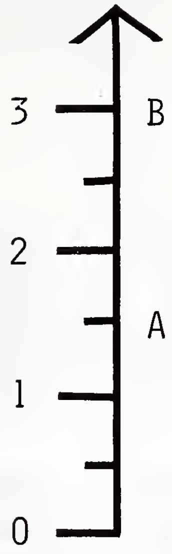
Instead of using horizontal lines Ed could have used a vertical line with interval markers to show the same thing:


15
10
Number of Papers
5
0

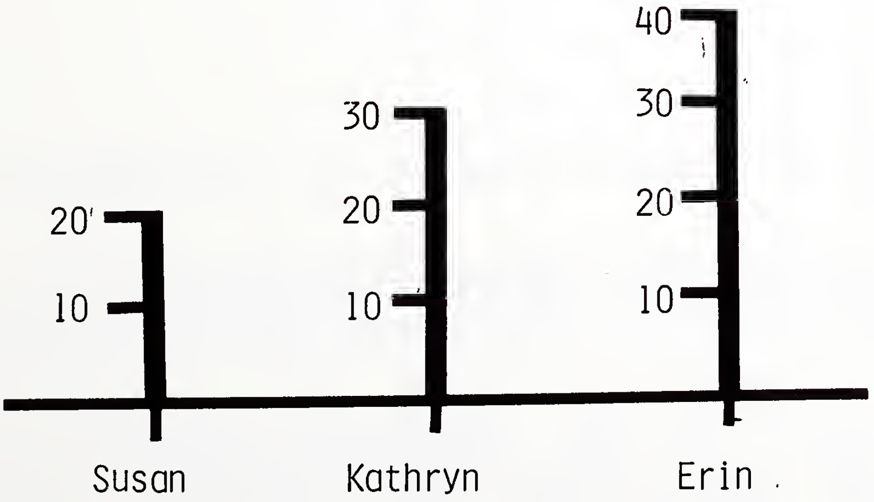
Find the letter A on the following number line. What number is A above? [p.57]



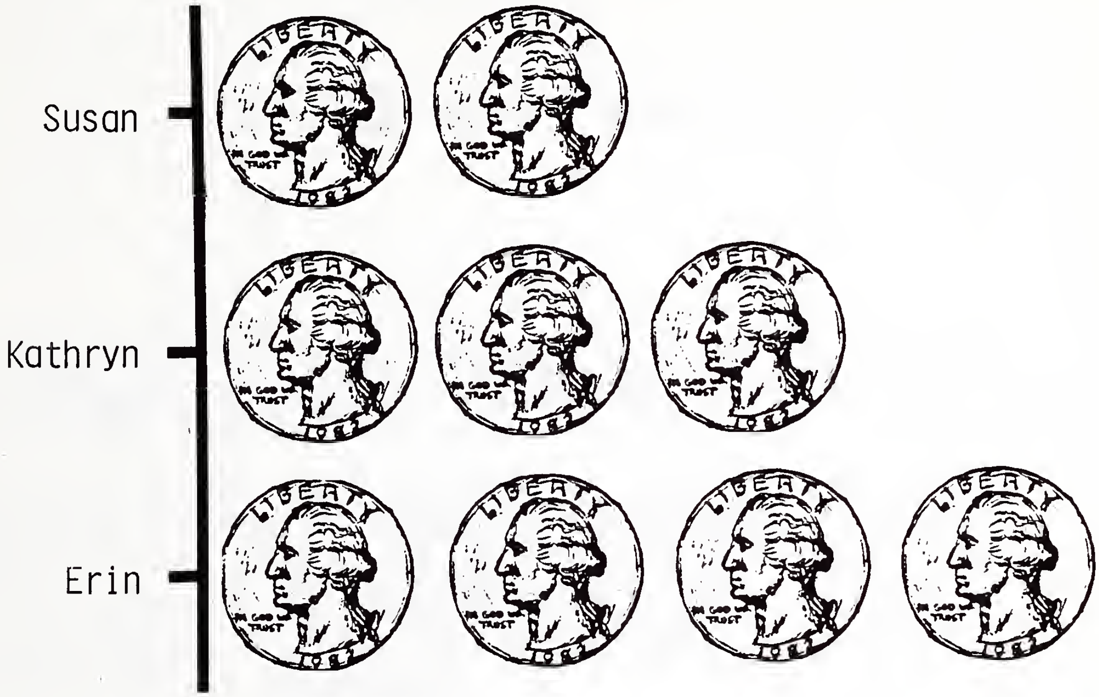
Look at this number line. What number do you get if you subtract A from B?



Westport Middle School had a contest to see who could collect the most papers. Kathryn, Susan, and Erin collected the most papers of all the children in the school. Each child kept a record using lines with interval markers. How many papers are represented by each interval? How many papers did Susan collect? Who collected the most papers? Who collected the least? [p.58]



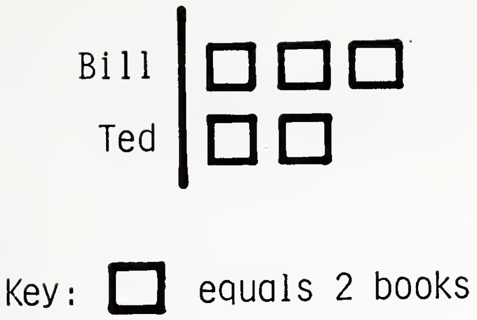
At the end of the paper drive, Kathryn, Susan, and Erin took their papers to the collection center. The money they received for the papers would be given to the school. They decided to make a graph of the money they collected to see who earned the most. For each quarter earned they put a picture of a coin next to their name on a piece of paper. Their graph looked like this: [p.59]



Who earned the most money? Who earned the least? How much money did Erin earn?

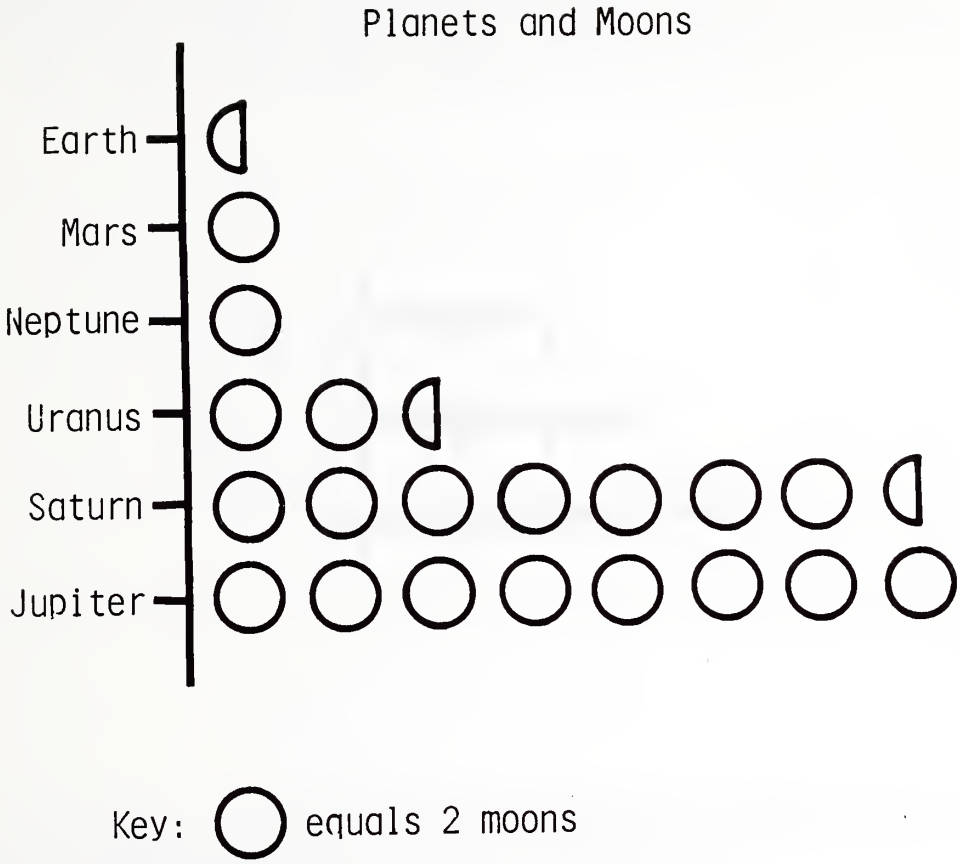
This kind of graph is called a pictograph. A pictograph is a special kind of graph that uses pictures to tell you something. In the preceding pictograph, each picture of a coin represents 25¢ or one quarter.

Here's another example of a pictograph. Bill and Ted joined a book club. After one month they made a graph of the number of books each read. For every two books read a picture like this was placed next to their name. The pictograph looked like this: [p.60]



How many books did Bill read? How many did Ted read? Always remember to look at the key for a pictograph. It will tell you what each picture represents.

Here's one more example of a pictograph: Anita often dreamed that she was in a spaceship traveling to the moon. One day, [p.61] Juan asked Anita which moon she was planning to visit. Juan reminded her that many of the planets have moons. Anita decided to make a pictograph showing the number of moons that belong to some of the planets. Be sure to look at the key first to find out what each picture represents.

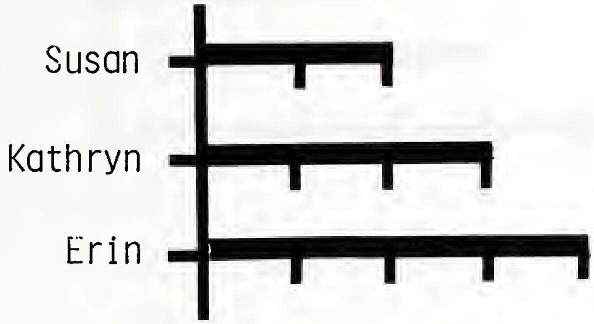


Notice the title at the top of the graph. A title tells you what the graph is about. The title of this graph is "Planets and Moons." Which planets have two moons? How many moons does Earth have? [p.62]

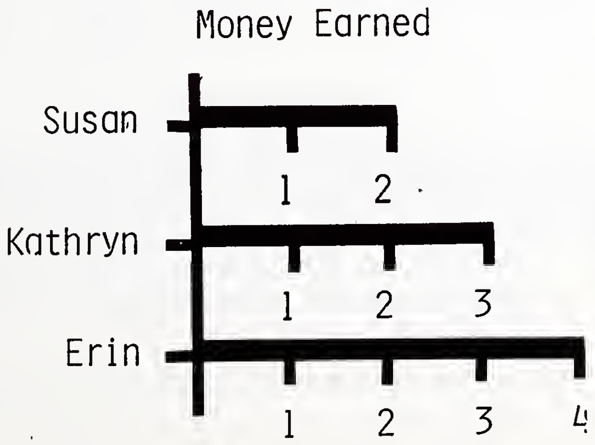
Notice that the moon picture for the Earth is cut in half. Since a whole moon picture represents two moons, a half of a moon picture represents one moon. How many moons does Saturn have?

In a pictograph, the pictures are also called symbols. A symbol is something that stands for or represents some other thing. In this example the ○ is a moon symbol. In an earlier pictograph, a □ was used as a symbol to represent a book.

Now let's go back to Kathryn, Susan, and Erin. Remember they made a pictograph to show who made the most money for the paper drive. Each coin symbol or picture represented a quarter. Instead ofpictures they could have used marks on a line to show the same thing. A mark could be put on the line for each coin symbol. Each mark would then represent one quarter. [p.63]

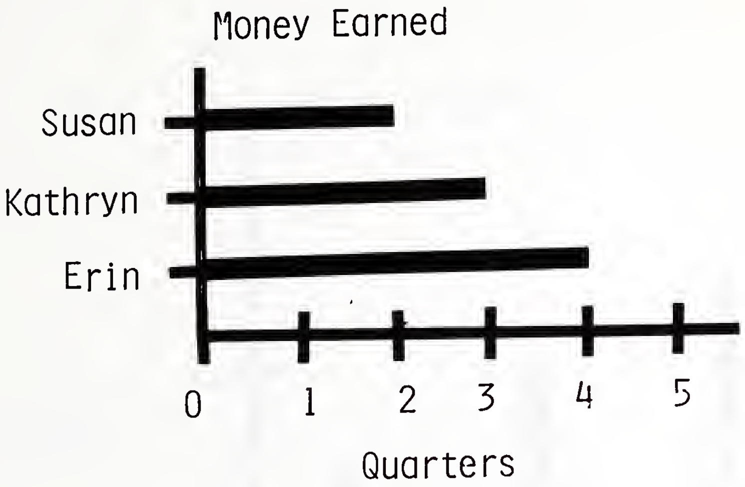


It would be easier to tell how many quarters each earned if a number were placed below each mark, like this: [p.64]



Each student now has a number line next to her name. Each number line shows how many quarters that student earned.

Putting numbers under each line takes up a lot of space. Here is a simpler way to show the same thing: [p.65]



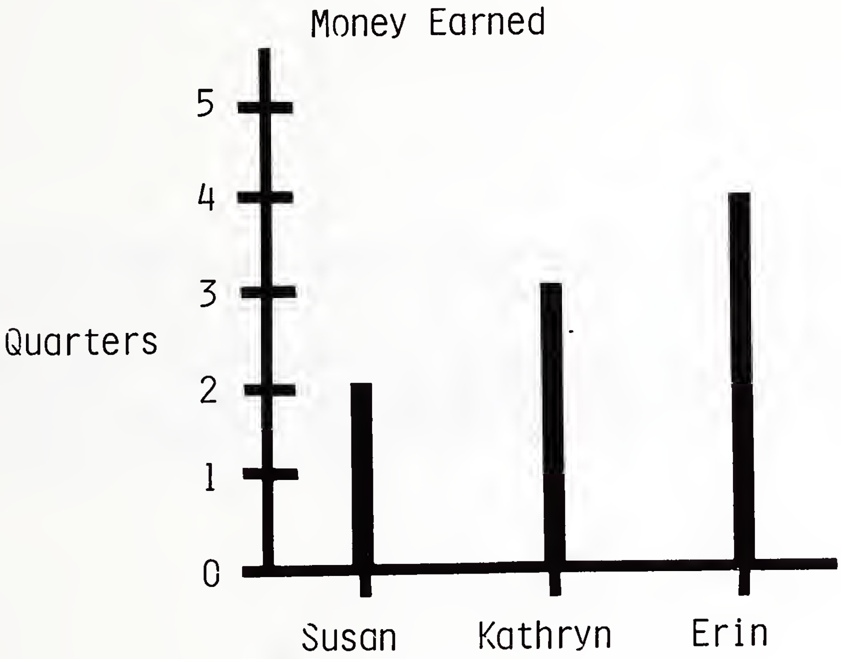
By putting a number line at the bottom of the graph we can remove the numbers from the other lines. To find out how much each student earned, just compare each line or bar with the number line at the bottom. For example, to find out how much money Susan earned, follow the line or bar next to her name until you reach the end. Then move your finger straight down to the number line and read the number. This is how much she earned. Now do the same thing for Kathryn and Erin.

TEACHER NOTE

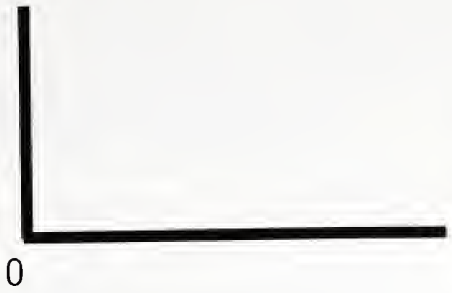
Have the students use both index fingers when comparing a bar with the numbered axis. One finger should be kept on the end of the bar for a reference point, while the other finger is moved in a straight line to the axis. This procedure should be followed with all bar graphs.

The graph you have just been examining is called a bar graph. Can you guess why? [p.66]

Since the lines or bars are horizontal, this graph is called a horizontal bar graph. We could show the same information by making a vertical bar graph, like this:

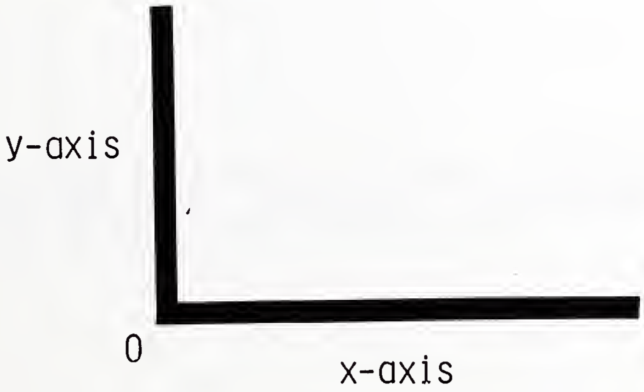


Now let's look more closely at the parts of a bar graph. A bar graph always has two lines that intersect each other. One is a vertical line and the other is a horizontal line. They are always perpendicular to each other, as shown in the following drawing: [p.67]



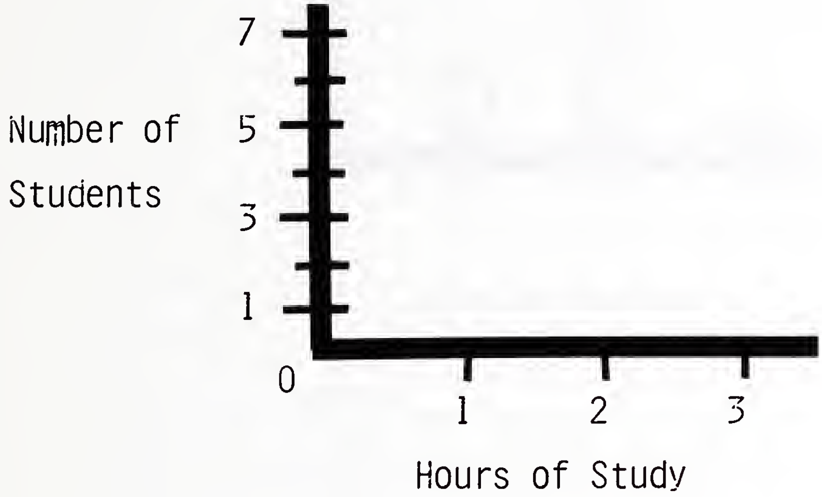
Notice the point where the two lines meet or intersect. This point is called the origin. The origin is marked with a zero.

The two lines that intersect in a graph are called axes. The horizontal line is called the horizontal axis or x-axis. The vertical line is called the vertical axis or y-axis. Notice that the plural of axis is axes. [p.68]

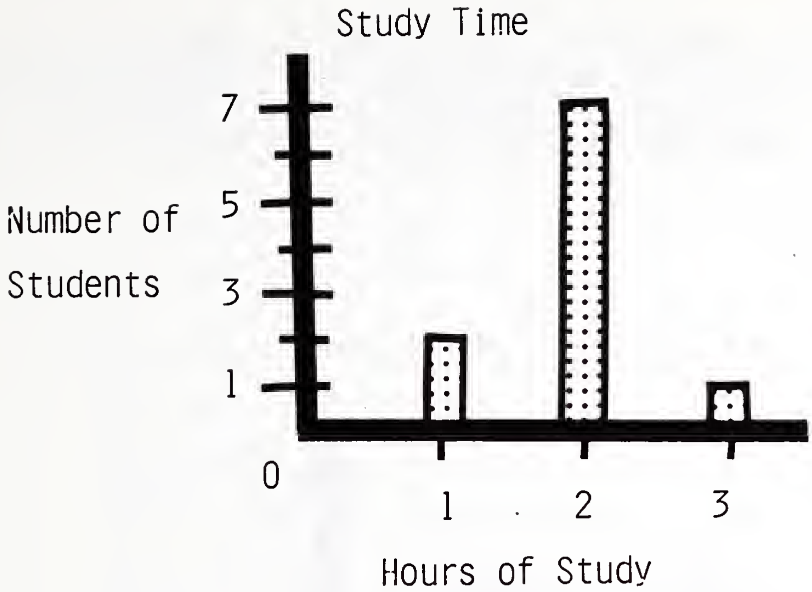


We can draw markers on the axes and put numbers next to them to indicate the amount of something. By doing this, we turn the axes into number lines.

Suppose Mr. Stone asked the students in his seventh grade class to tell how many hours they spent each night studying. Of the 10 students in his class, two said they studied one hour each night, seven said they studied two hours, and one said she studied three hours. Mr. Stone decided he would make a bar graph. On the x-axis he put marks to show the number of hours spent studying. On the y-axis he put marks to show the number of students. He also put a label near each axis. A label makes it easier to tell what each axis represents. [p.69]



Now Mr. Stone put three bars on the graph. The bars showed how many students spent one hour, two hours, and three hours studying. He also put a title at the top of the graph.



Find the bar for one hour of study and trace it up the page until it stops. Then move your finger straight across to the vertical axis to find out how many students study for one hour each night. Do the same thing for the other two bars.

Notice that the numbers on the axes are smaller near the origin and larger farther away from the origin. This is the way graphs are always made. On the vertical axis, the numbers increase as they go up the page. On the horizontal axis, the numbers increase as they go across the page to the right. The reason for this is [p.71] that you are moving farther and farther away from zero, the origin.

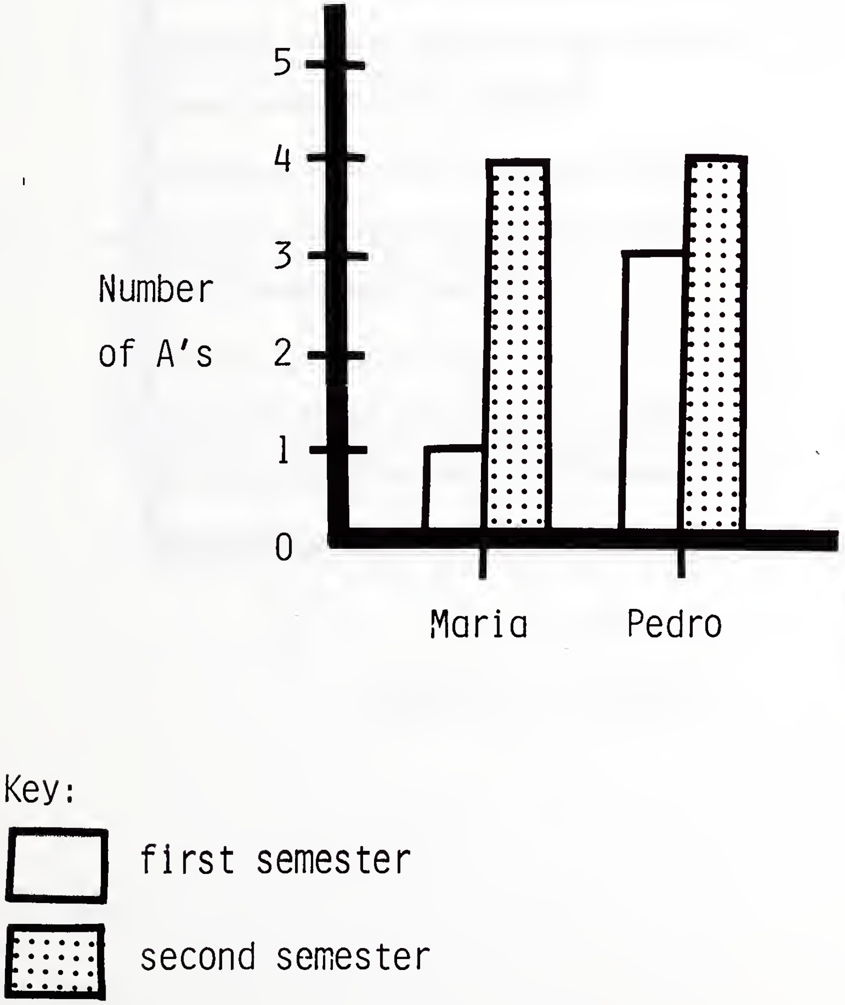
You should also notice that not every marker on the y-axis has a number next to it. It is not necessary to number each marker on an axis. In this graph, for example, there is a marker numbered 3 and a marker numbered 5. The marker in between these two is not numbered. What number is missing?

Here is one more thing you should have noticed about the bars in a bar graph: The bars are always parallel to one of the axes and perpendicular to the other one. Which axis are the bars parallel to in the graph you just examined? Which axis are they perpendicular to?

Sometimes bars are grouped in pairs in a bar graph. Here is an example: Pedro and Maria were arguing about who improved the most in school from the first to the second semester. To solve the argument, they drew a bar graph showing the number of A's each received.

[p.72]

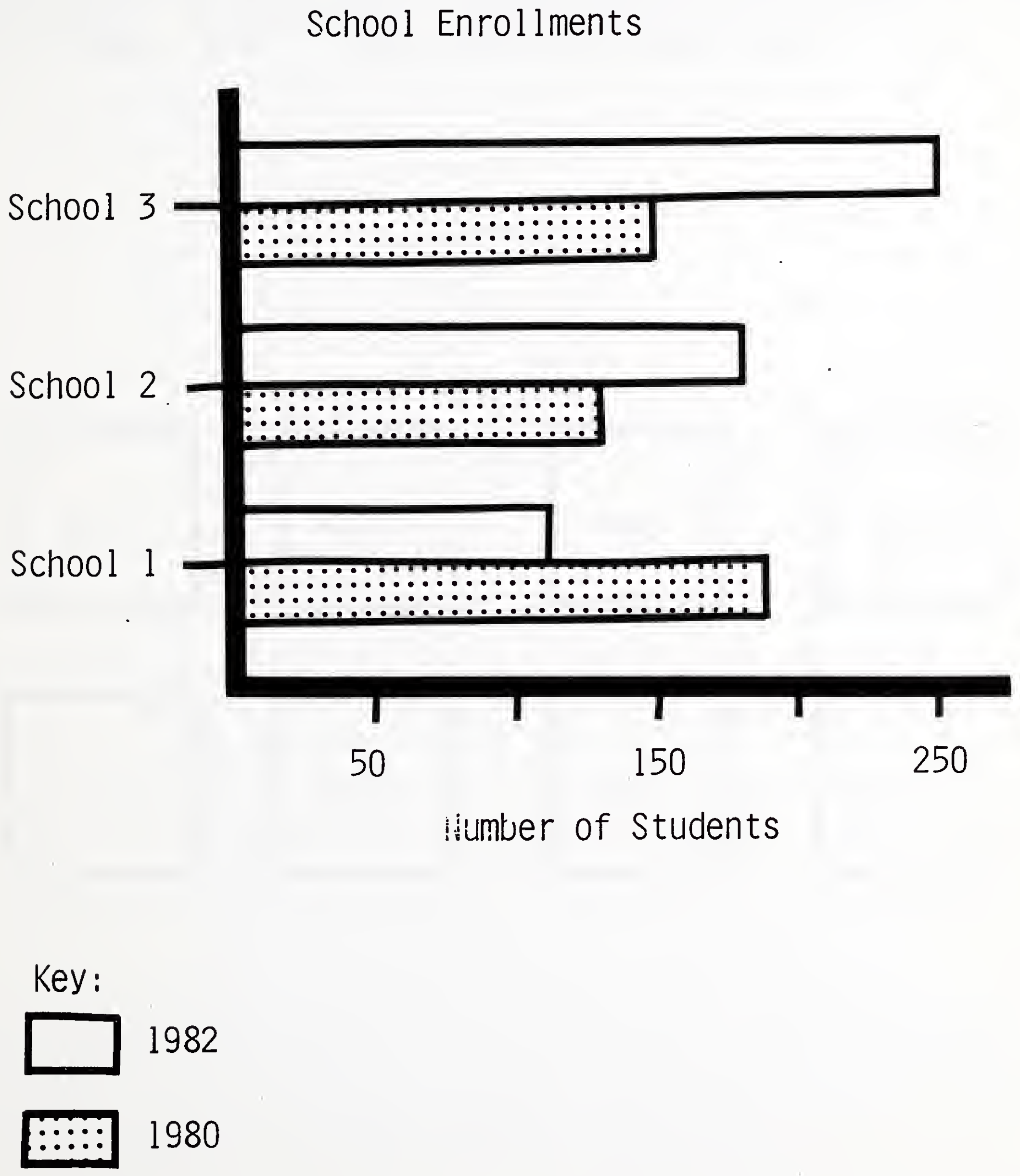
A's Received by Maria and Pedro



Notice that there are two bars for each student. One bar is for the first semester and the other bar is for the second semester. The key tells you which bar is used for each semester.

Who improved the most, Pedro or Maria?

Here is another example of a double bar graph. It shows the number of students enrolled at three schools in 1980 and 1982. [p.73]

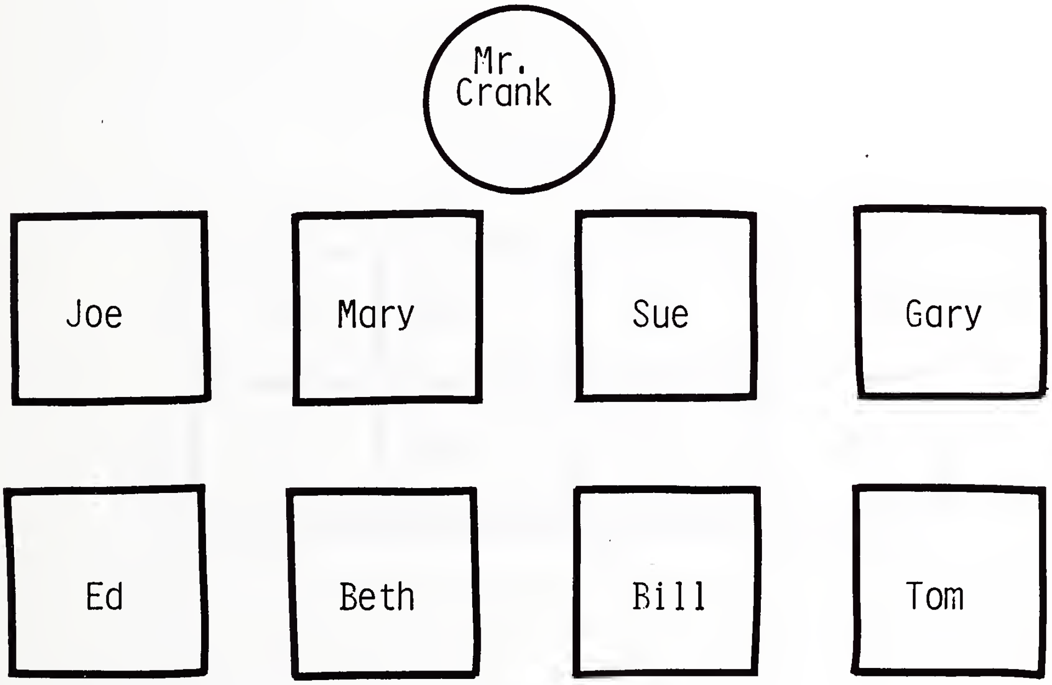


At which school did the number of students increase the most from 1980 to 1982? [p.74]

Which school had less students in 1982 than in 1980?

It was the first day of school. Eight students were just beginning 6th grade. They reported to their new classroom and met their teacher, a man named Mr. Crank. Mr. Crank assigned a desk to each student. He drew a map to help them remember where everybody's desk was located. On the map he drew a square to represent each desk and then put the name of each student in one of the squares. At the top of the map, he put a circle to represent his desk. This is what the map looked like:

[p.75]



From the map you can see that the desks are arranged in rows and columns. A row is a horizontal arrangement of things. On the map there are two rows of desks. A column is a vertical arrangement of things. On the map there are four columns of desks.

The first row of desks is the one nearest to Mr. Crank's desk. Who are the students in the first row? How many students are in the second row? [p.76]

The first column of desks belong to Joe and Ed. Which students are in the third column? How many students are in each column?

Which student's desk is in the fourth column, second row? Which student's desk is in the first column, first row?

Instead of a map, Mr. Crank could have drawn a table. A table is a way of arranging information by using rows and columns. Here is how Mr. Crank's seating arrangement would look in a table: [p.77]

		Column			
		1	2	3	4
Row	1	Joe	Mary	Sue	Gary
	2	Ed	Beth	Bill	Tom

Another example of a table is a bus schedule. The table below tells you when each of three buses arrives at three different streets. The three buses make up the rows and the three streets make up the columns. So, for example, the table tells you that the second bus arrives at Deer Avenue at 8:30.

[p.78]

Bus Schedule

	Oak St.	Plum St.	Deer Ave.
First Bus	7:30	8:05	8:15
Second Bus	7:35	8:00	8:30
Third Bus	7:45	8:20	8:40

Tony rides the bus to school. He gets on at Oak Street. His school is at Plum Road. Suppose Tony takes the first bus. What time will the bus pick him up? When will he get to school?

School starts at 8:15. If Tony takes the third bus, will he be late?

How many minutes does it take the first bus to get from Oak Street to Deer Avenue? [p.79]

Let's try one more table. This one shows how many sit-ups and push-ups three students did over two days.

Exercise Record

	First Day		Second Day	
	Push-ups	Sit-ups	Push-ups	Sit-ups
Bill	10	18	11	18
Gus	8	20	10	23
Anna	12	19	12	20

How many sit-ups did Gus do the first day?

How many push-ups did Anna do the second day? [p.80]

Who did more push-ups on the second day, Gus or Anna?

Find out how many more sit-ups each student did on the second day than on the first. Then decide who improved the most.

Here is a table showing the number of TV shows watched by four students one week. [p.81]

Students	Shows
Todd	7
Meg	12
Jon	13
Juanita	4

See if you can make a bar graph from the information in this table.

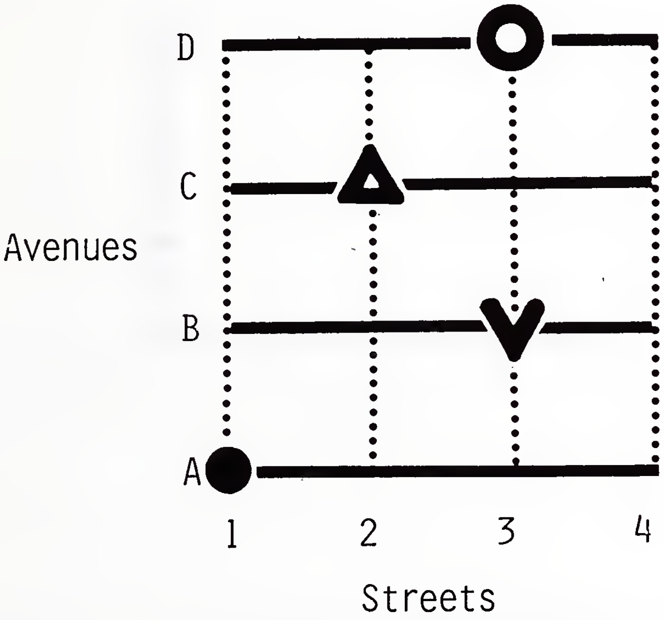
TEACHER NOTE

Participation Exercise: Using the graph paper with either a one inch or half inch grid, have the student use these data to construct both a horizontal and a vertical bar graph. (If the one inch grid is used, each interval must be used to represent more than one TV show.) Form the bars by either lining up several pins or by stretching a rubber band between two pins. Have the student label both axes with a slate and stylus or a braillewriter.

Participation Exercise: Have the student construct a bar graph for data of his/her own choosing. For example, the number of hours spent each week engaging in various activities, the heights of family members, the amount of money spent each week on different things.

The following is a map of part of a city. At the lower left-hand corner of the map is a police station, which is represented by a dot. This police station is responsible for patrolling the roads shown on the map. The roads drawn with a dotted line are called streets. The roads drawn with a solid line are called [p.82] avenues. The police station is located at the corner of 1st Street and Avenue A. You can think of the streets as colums and the avenues as rows.

Patrol Map



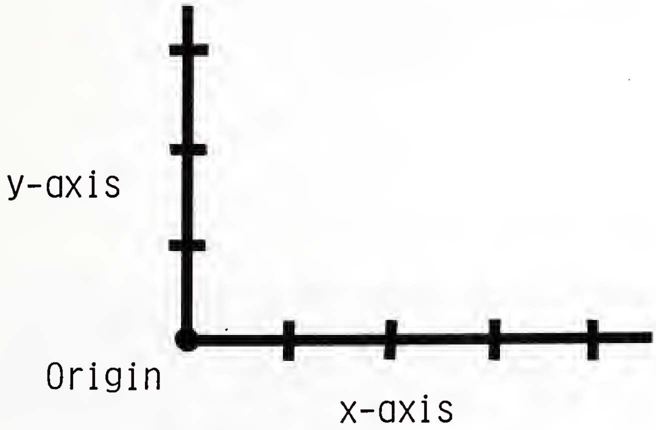
A store at the intersection of 3rd Street and Avenue D has just been robbed. See if you can find the store on the map.

The store is represented by a circle on the map. If you did not [p.83] find the circle, the robbers got away. To get to the store, the police might drive two blocks over to 3rd Street and then three blocks up to Avenue D.

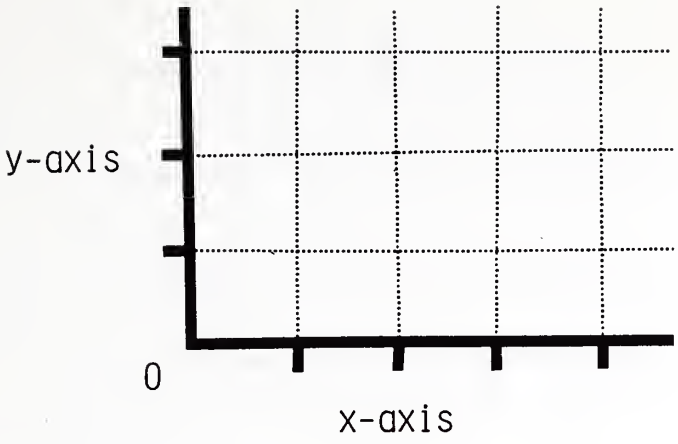
Find the triangle on the map. This is the hideout of the robbers. Where is it located? How many blocks over is it from the police station? How many blocks up is it from the police station?

Now let's return to graphs. Remember that a graph has an x-axis and a y-axis that intersect at a point called the origin. Interval markers are placed along the axes to indicate the amount of something.

[p.84]



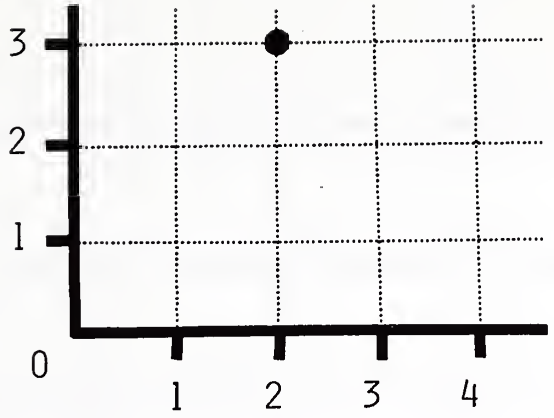
If the interval markers on the axes were extended, they would form a series of boxes, called a grid. A grid looks something like our map of streets and avenues.



You can think of a grid as being made up of rows and columns. The rows start at the y-axis and go across the page. The columns start at the x-axis and go up the page. There is one row for each interval marker on the y-axis. There is one column for each interval marker on the x-axis. [p.85]

The grid makes it easy to find places on a graph. If you wanted to locate a particular point on a graph, you would need two numbers or values. You need a value for x and a value for y. For example, if you wanted to find a point located where x = 2 and y = 3, you would start at the origin and move 2 interval markers to the right along the x-axis and then three squares up. Try this on the graph shown below.

[p.86]



The x and y values used for locating specific points on a graph are called coordinate values. Instead of writing x = 2 and y = 3, coordinate values are usually put in parentheses. The first number in the parentheses is the value for x. The second number is the value for y. So x = 2 and y = 3 would be written as (2, 3). x = 4 and y = 6 would be written as (4, 6).

Determine the value of x for each of the following coordinate pairs:

(2, 3), (4, 8), (5, 1), (1, 9), (0, 3)

Determine the value of y for each of the following coordinate pairs: [p.87]

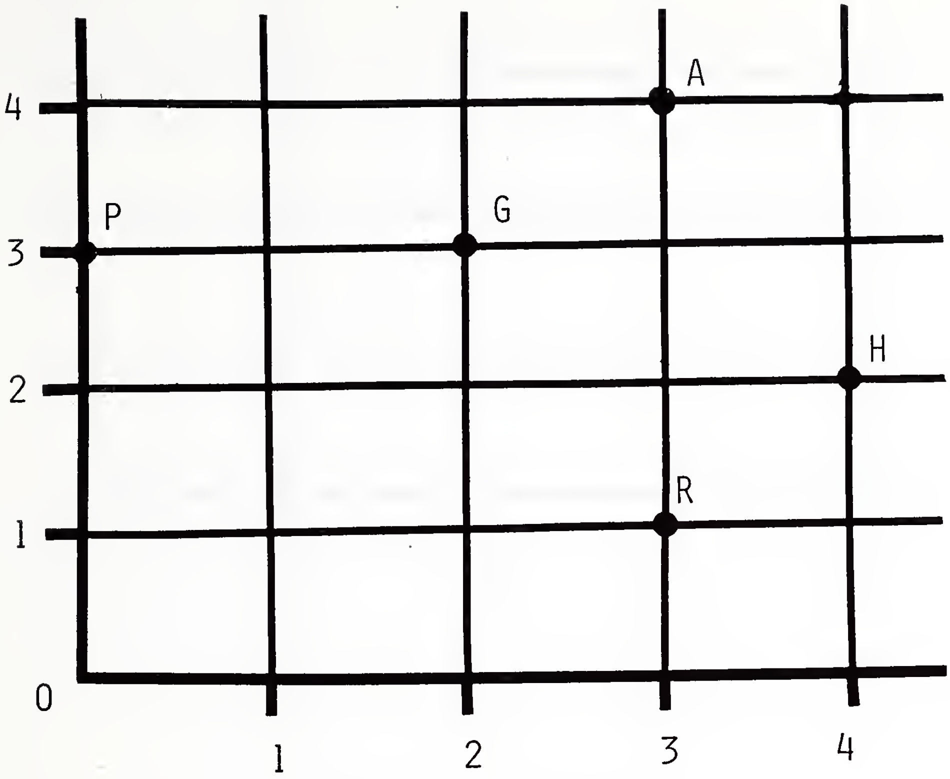
(2, 3), (5, 1), (6, 3), (9, 0), (5, 4)

Locate the point on the graph for each of the coordinate pairs in the following list. When you find each point on the graph there will be a letter located next to it. If you do it correctly, the letters will form a word. See if you can break the secret code and find out what the word is.

Here are the coordinate pairs:

(2, 3), (3, 1), (3, 4), (0, 3), (4, 2)

[p.88]

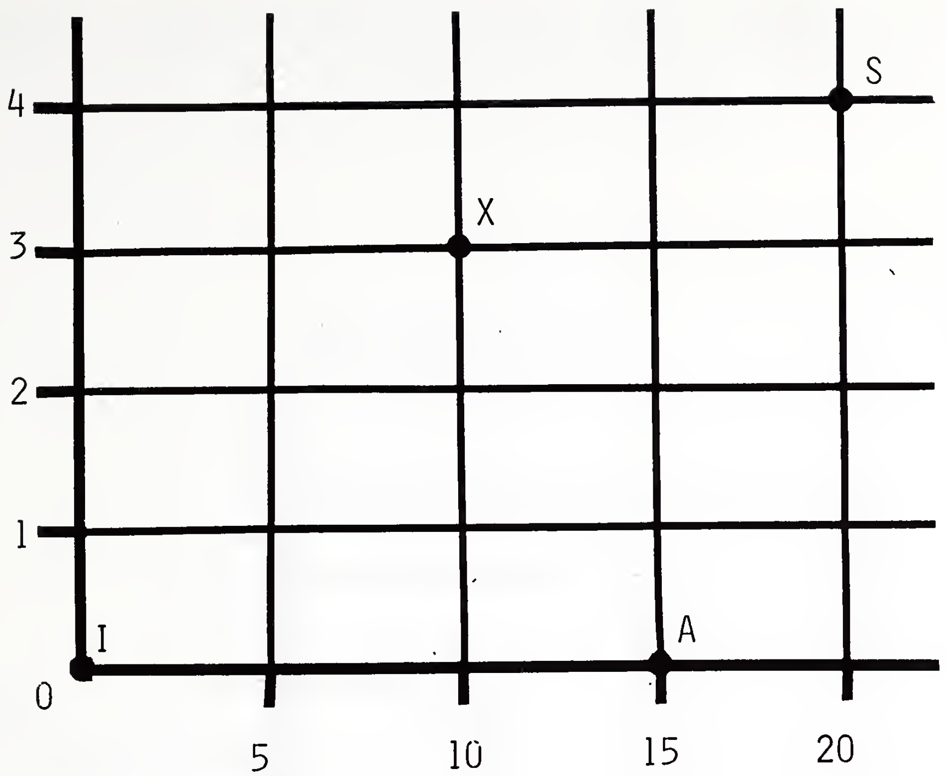


4
3
2
1
0
1
2
3
4
A
P
G
H
R

[p.89]

Here is another hidden word problem. Determine what word is formed by these coordinate pairs:

(15, 0), (10, 3), (0, 0), (20, 4)

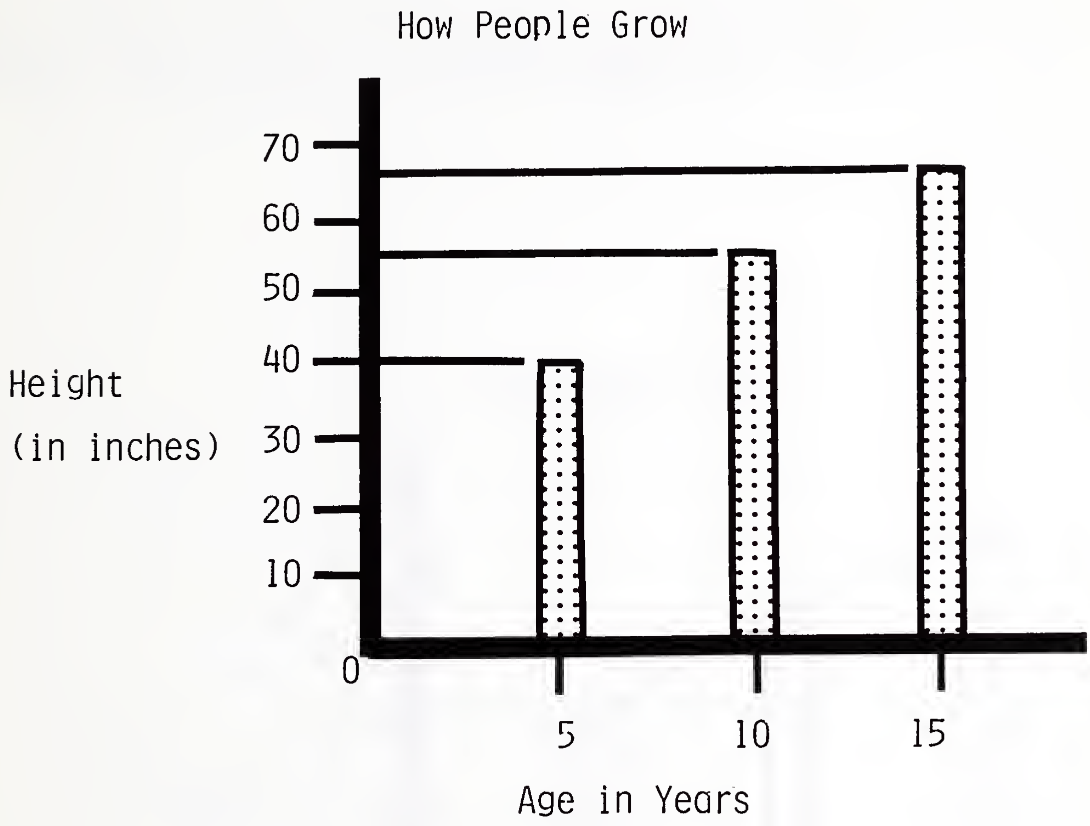


TEACHER NOTE

Participation Exercise: The purpose of the following game is to provide practice with coordinate pairs. Using the graph paper with a one inch grid, have the student position pins at the following coordinate locations: (2, 1), (2, 3), (4, 3), 4, 1). Connect the pins with rubber bands and see if the student can recognize the resulting shape (in this case, a square). Here are the pairs for a triangle: (1, 1), (2, 3), (4, 1); for a rectangle: (1, 6), (3, 6), (3, 2), (1, 2); for the side view of a chair: connect (2, 0) to (2, 2) to (2, 4), then connect (2, 2) to (4, 2) to (4, 0). Feel free to construct your own shapes.

As you grow older, you also grow taller. This is shown in the bar graph below. Notice the title at the top of the graph. Remember that a title tells you what the graph is about.

[p.90]



What is the label for the x-axis? What is the label for the y-axis?

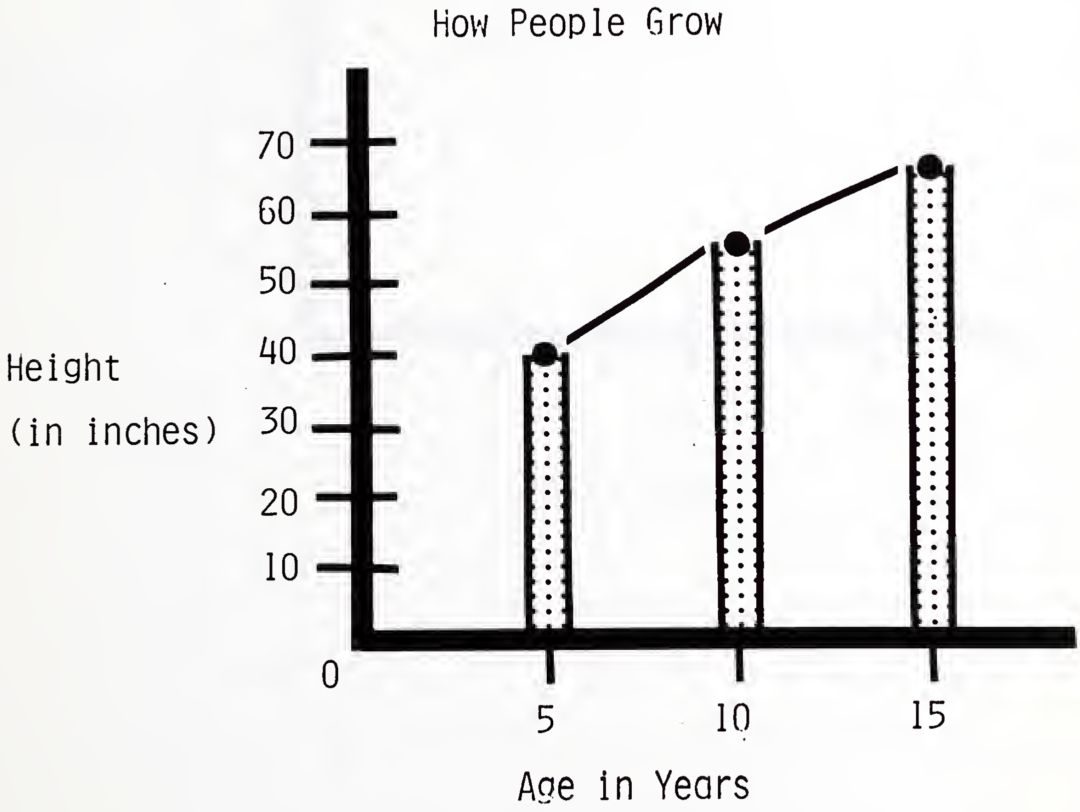
According to the graph, how tall do children usually grow by the age of 5?

Examine the bar for age 10 carefully. Go to the top of this bar and then move your finger along the line that connects the top of the bar with the vertical axis. Your finger will come to a point on the vertical axis that is halfway between the interval markers for 50 and 60 inches. Fifty-five is halfway between 50 and 60. So the graph tells you that children are usually 55 inches tall by age 10. [p.91]

How many inches do children usually grow from age 5 to age 15?

Now let's change the graph a little. At the top of every bar we are going to put a dot. We are also going to connect the dots with lines. Here is how our graph looks now:

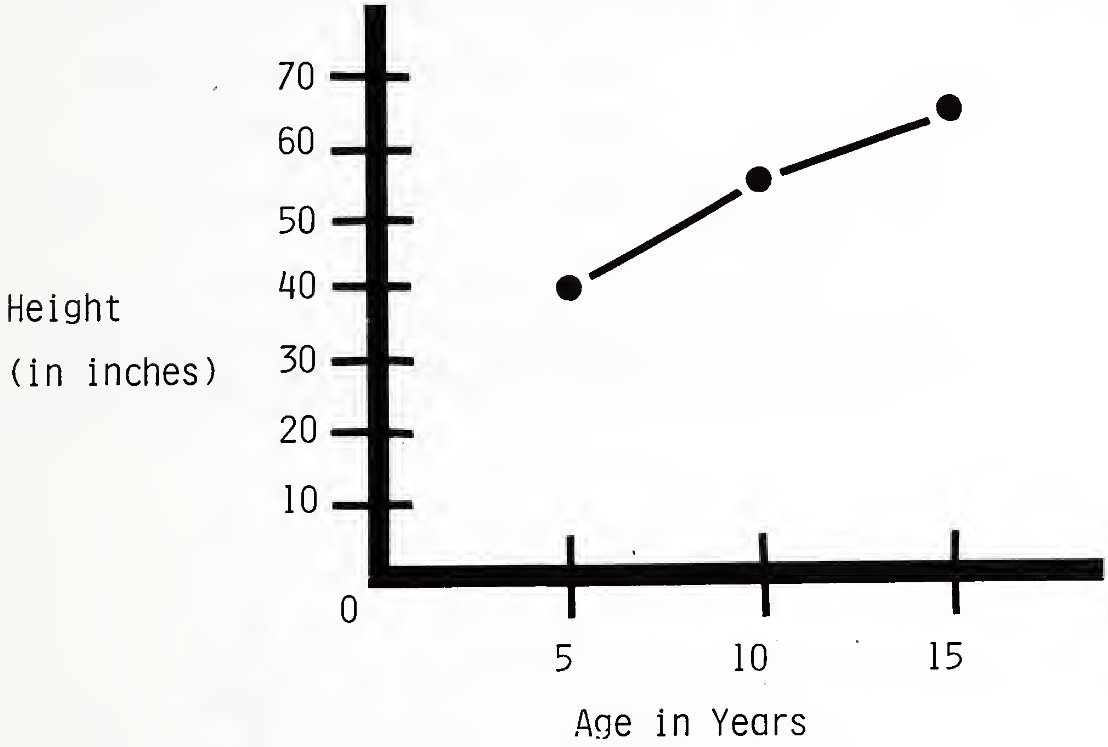
[p.92]



Trace the line from dot to dot. What do you notice about the line?

The line goes diagonally up the page. As the bars get taller, the line gets higher. Since the line gives you the same information as the bars, the bars can be dropped from the graph. Now the graph looks like this: [p.93]

How People Grow



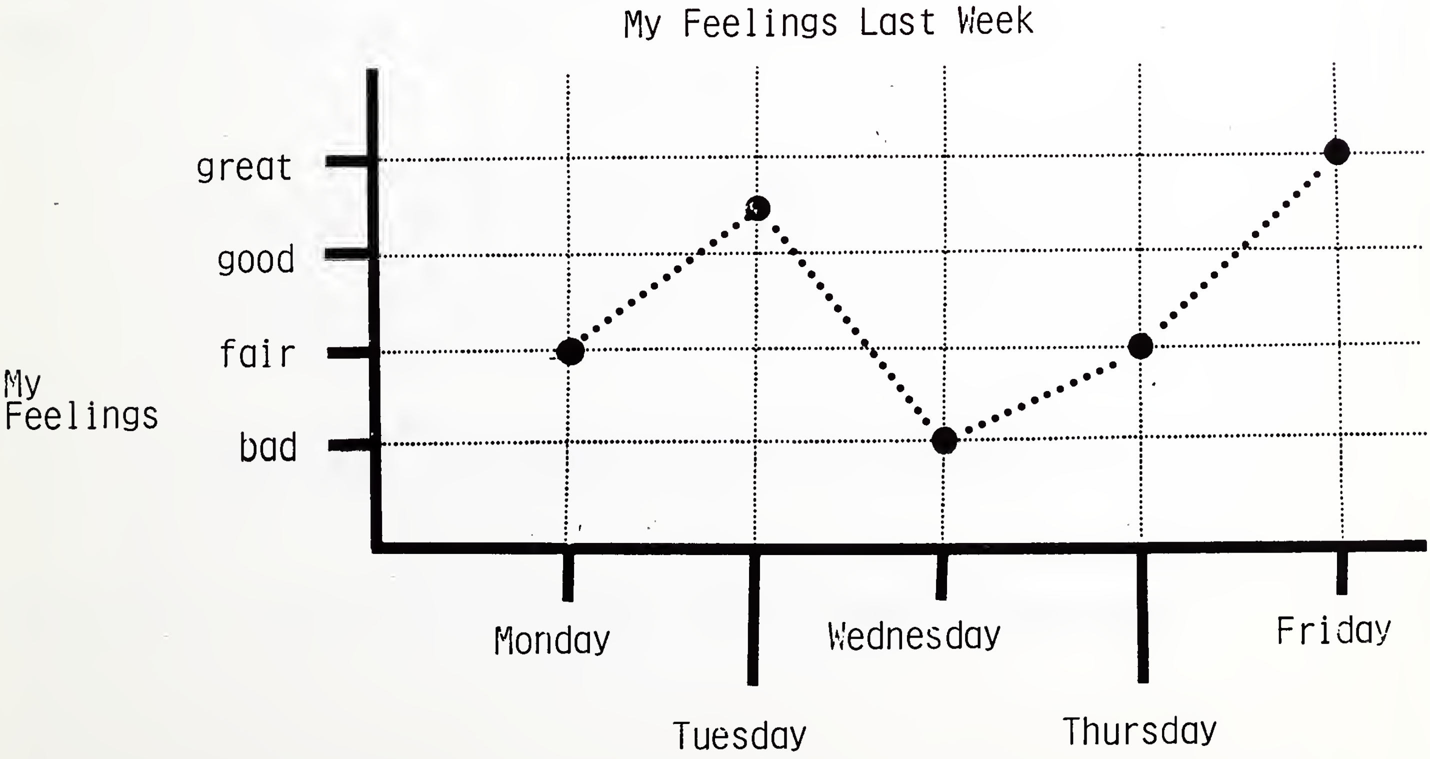
This kind of graph is called a line graph. You might think of a line graph as a graph with invisible bars. [p.94]

Whenever you come across a line graph, go through the following steps:

1. Read the title. This will tell you what the graph is about.
2. Locate the x and y axes. This will tell you how big the graph is.
3. Read the labels next to the axes. These will tell you what each axis represents on that graph.
4. Examine the numbers along the x and y axes. Determine how many units are represented by each interval. See how large the numbers get on each axis.
5. Read the key if there is one.
6. Examine the line on the graph.

Study these steps until you can repeat them from memory.

Line graphs are a good way of showing how things change. Sheri drew a line graph to show how her feelings changed over the week. Her graph looked like this: [p.95]



Notice that this line graph has a grid. A grid makes it easier for you to interpret a graph. For example, suppose you wanted to know how Sheri felt on Friday. To find out, you first locate Friday on the horizontal axis. Then trace the grid line up the page until you feel the dot on the line. Now trace the horizontal grid line to the left until you reach the vertical axis. There you will find the word great. So the graph tells you that Sheri felt great on Friday. [p.96]

TEACHER NOTE

The student should use the following procedure for locating the y value that is associated with a particular x value: (1) keeping the left index finger on the x value of interest as a reference point, move the right index finger up the grid line until the data curve is reached, (2) put the left index finger next to the right one, (3) keeping the right index finger in place on the data curve as a reference, move the left index finger to the y-axis. Reverse this procedure for locating the x value that is associated with a particular y value.

On Monday Sheri felt fair. On what other day did she feel fair?

On what day did she feel bad?

On which day did Sheri feel better, Monday or Tuesday?

How did Sheri's feelings change from Wednesday to Friday?

How did she feel on Tuesday?

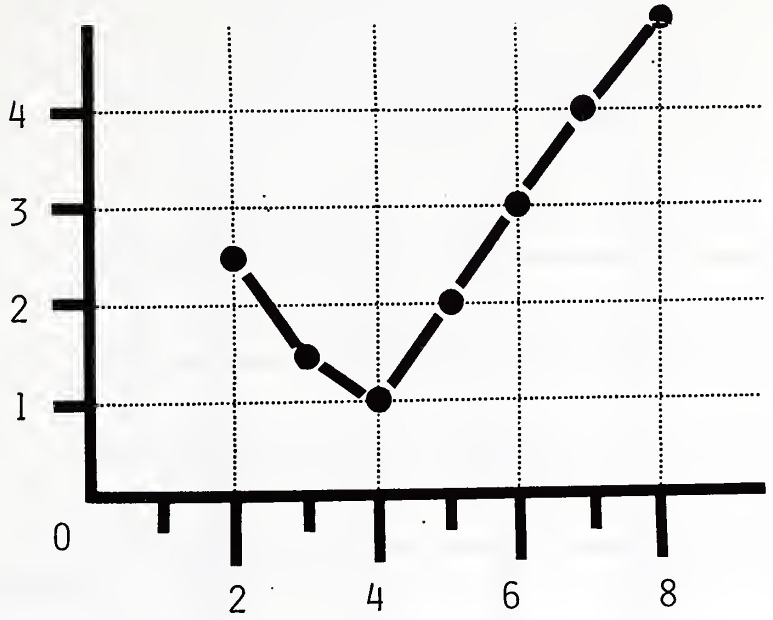
TEACHER NOTE

Explain to the student the need to interpolate when a data point falls between two interval markers. When this occurs, the student must (1) determine the possition of the data point with respect to the grid lines that bracket it, (2) follow the nearest grid line to the axis, (3) locate the interval markers that bracket the point, and (4) calculate the interpolated value.

In this example, the data point for Tuesday is halfway between "good" and "great" on the y-axis, so Sheri's feelings on Tuesday fall midway between these descriptions.

Did Sheri's feelings rise or fall from Tuesday to Wednesday? [p.97]

Examine the following line graph. Determine the coordinate pairs for the highest and lowest points of the line.

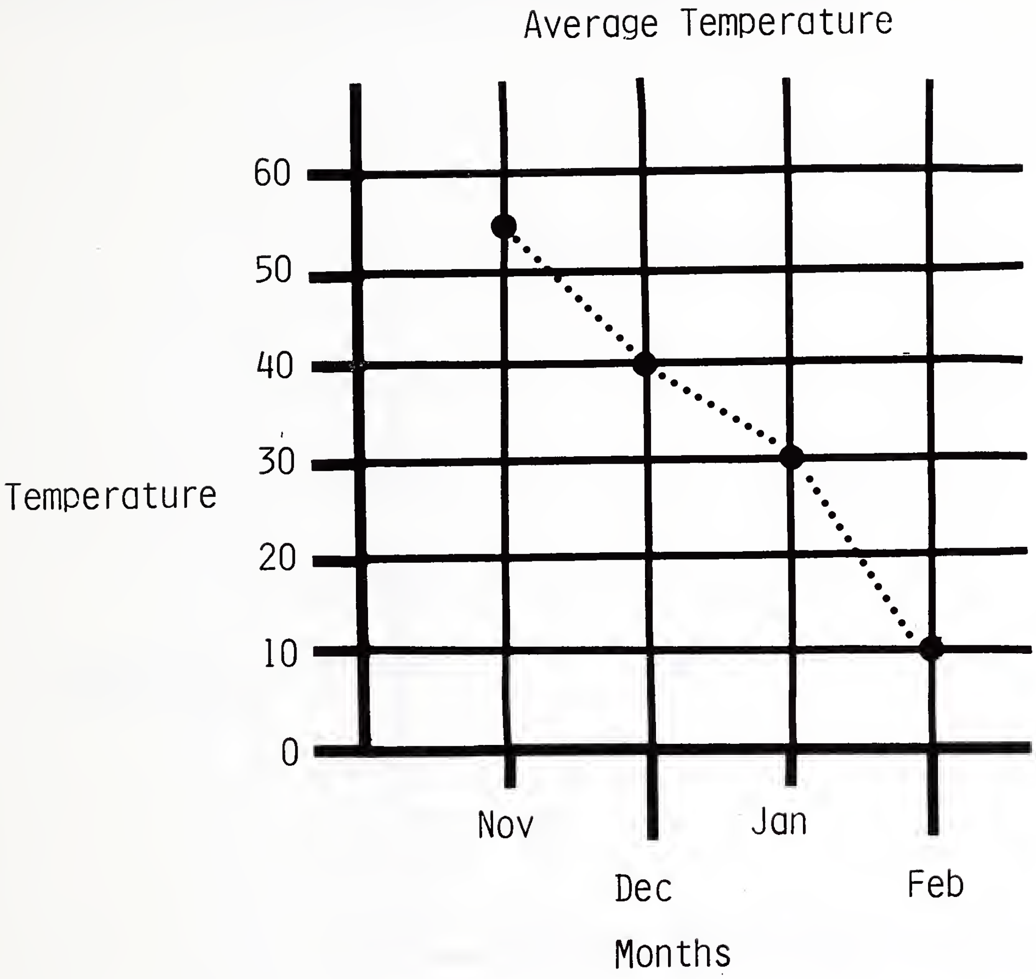


TEACHER NOTE

In order to determine the coordinate values of a point, the following procedure should be followed: (1) keeping the left index finger on the point as a reference, move the right index finger down to the x-axis and read the x value, (2) replace the left finger on the point with the right finger, (3) keeping the right finger in place, move the left finger to the y-axis and read the y value.

In this example, have the student pay special attention to the task of interpolation.

Here is another line graph. It shows how the temperature changed at noon over four months of the year. Determine the coordinate pairs for the highest and lowest points of the line. [p.98]



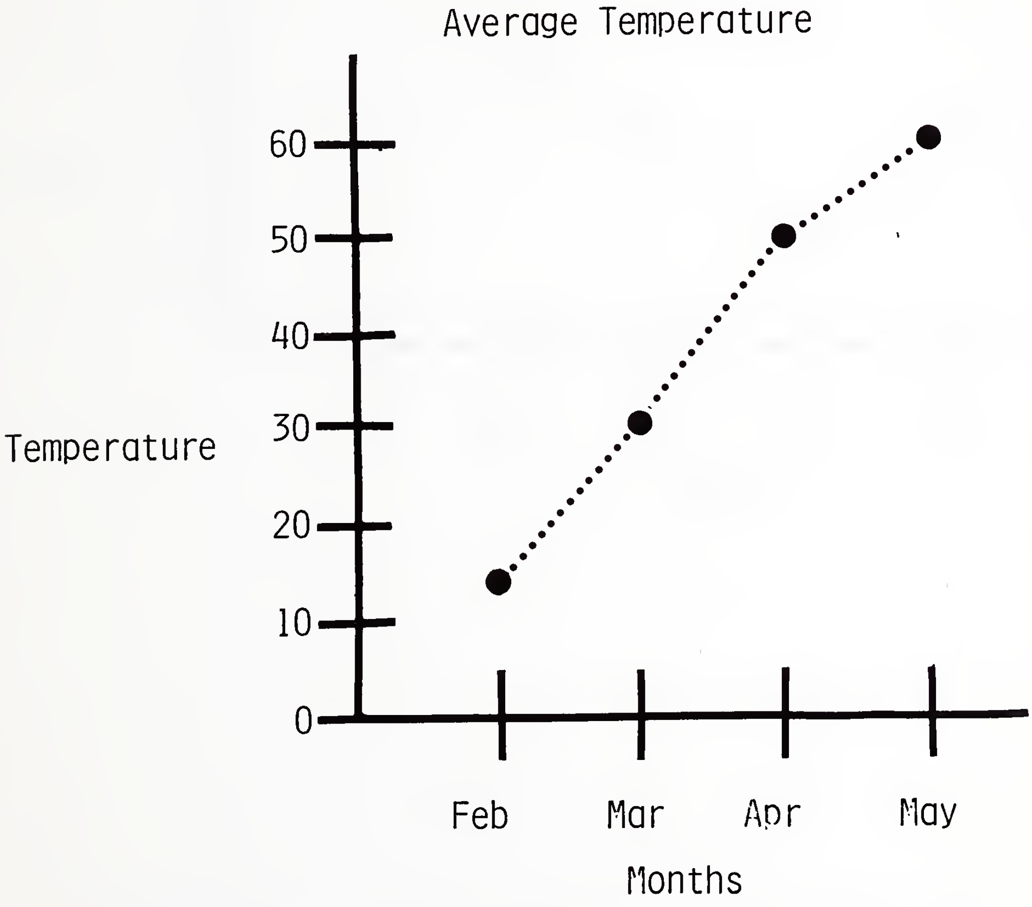
Notice that the line goes down from November to February. So the temperature went down or decreased from September to December. We call this a decreasing trend.

[p.99]

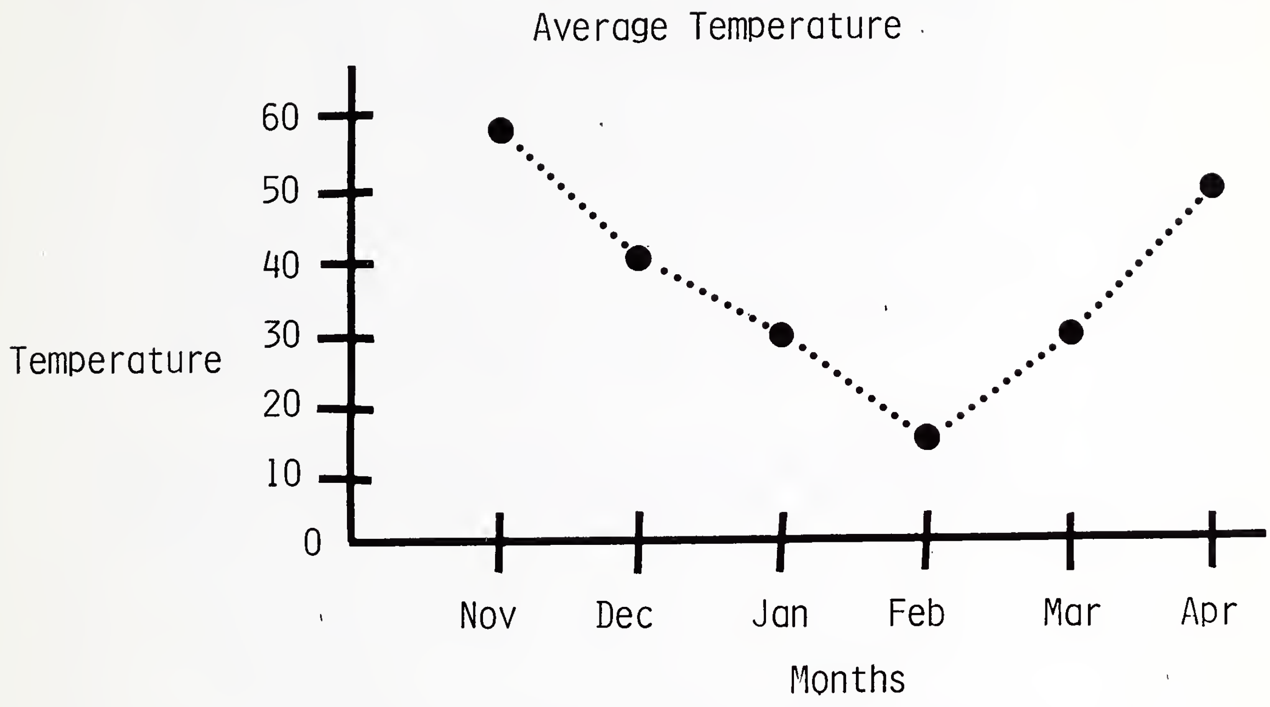
The direction a line goes is its trend. If a line goes down the page when you move your finger a long it from left to right, that line is said to have a decreasing trend.

If a line goes up the page when you move your finger along it from left to right, that line is said to have an increasing trend.

The following line graph shows an increasing trend. [p.100]



Here is a line graph that has both a decreasing trend and an increasing trend: [p.101]



What is the trend in temperature from November to February?

At what month does the trend change? [p.102]

During what months is there an increase in trend?

TEACHER NOTE

Participation Exercise: Have the student scan the display to get an idea of its overall dimensions, using the two-hand technique learned earlier in the program. Have the student locate the circle and square. Then say:

> "Now we're going to see how quickly you can get from the house to the cheese factory. Put your right finger on the circle. Put your left finger next to your right one. You will follow the path with both fingers. When I say go, try to find your way to the square. Go as quickly as you can and try not to be fooled by the dead end alleys. I'll time you with my (stop) watch. I'll also count the number of wrong turns you make. Then we'll do the same thing three more times. After each run, we'll mark down your time and errors. When we're all finished, we'll make a graph of your data. OK, ready, go." (Time run and count number of wrong turns made by lead finger. Have the student keep a written record of the time and error scores obtained on each run.)

When the four runs are completed, have the student participate in the construction of the graphs (using the graph paper, pins, cork board, and rubber bands provided with the program). Start with the speed scores.

x-axis (trials or runs)

--ask student:

- How many interval markers do you need? (4)
- How would you space the interval markers? (equal spacing)
- What label would you put under each interval marker? (1, 2, 3, 4)
- What label would you use to indicate what this axis represents? (runs, trials, attempts)

y-axis (time in seconds)

--say to student:

- Look at your time data. What was your longest time?
- How high does the y-axis have to be? (At least as high as the largest number of seconds)

- * How many interval markers do you need? (Explain that this is optional: 1 for each second if the times are short; 1 for every 2 seconds (or 5 seconds, or 10 seconds) if the times are longer. The y-axis should be about 2/3 as long as the x-axis).

- * How would you space the interval markers? (equal spacing)

- * What label would you put next to each interval marker? (0, 1, 2 . . .; 0, 2, 4 . . .; 0, 5, 10 . . .; 0, 10, 20 . . .)

- * What label would you use to indicate what this axis represents? (seconds, time)

Title

--say to student:

- * What title would you use for this graph? (descriptive, but short)

- * Where does the title go? (top of graph)

Have the student fill in all of the above information on the graph paper, using a slate and stylus or a braillewriter. Then position the graph paper on the cork pad. Have the student construct a bar graph of his/her speed scores. Start with trial 1. Have the student put one pin on the axis at the trial 1 interval marker, and another at the point corresponding to the speed attained on that trial. Then stretch a rubber band between the two pins. Proceed in the same way with trials 2 through 4. This completes the bar graph.

Ask appropriate questions. For example: What was your fastest (slowest) run? How much faster were you on trial 4 than trial 1?

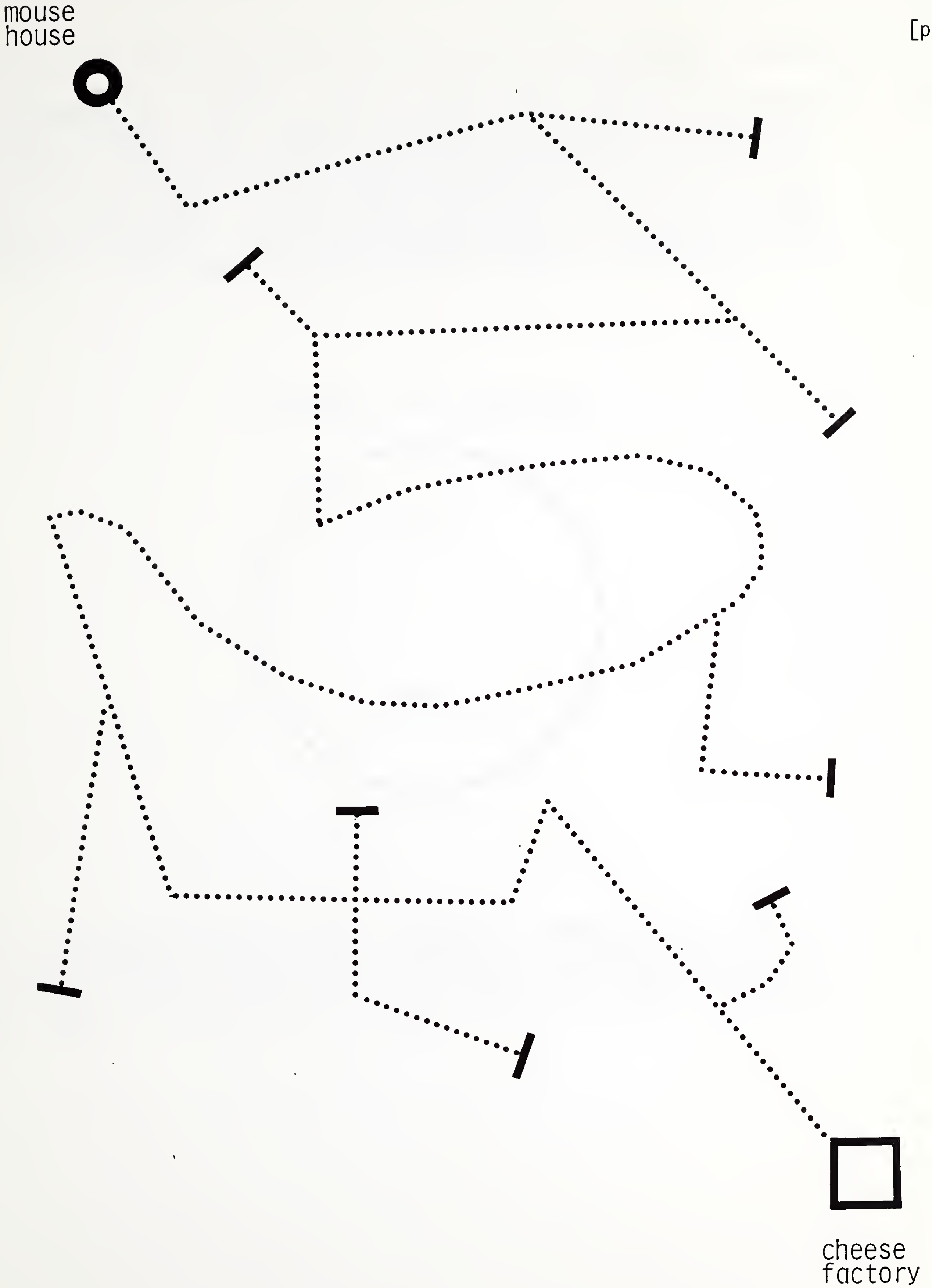
Then show the student how the bar graph can be converted to a line graph by connecting the tops of the bars. Remove the pins on the axis and stretch the rubber bands between the remaining pins. Ask appropriate questions. For example: What is the trend of the data over the four runs? If you made a fifth run, would you expect your speed to be greater than, less than, or equal to your speed on the fourth run? Between what two runs did you improve the most?

If you have more than one student, graph their data on separate sheets first and then combine them on the same sheet to form a multiple line graph. The rubber bands used

for one line can be twisted to distinguish it from the line made with the non-twisted rubber bands. Have the students compare the lines. Ask appropriate questions. For example: Who improved at a faster rate? Who was the slowest on the second run?

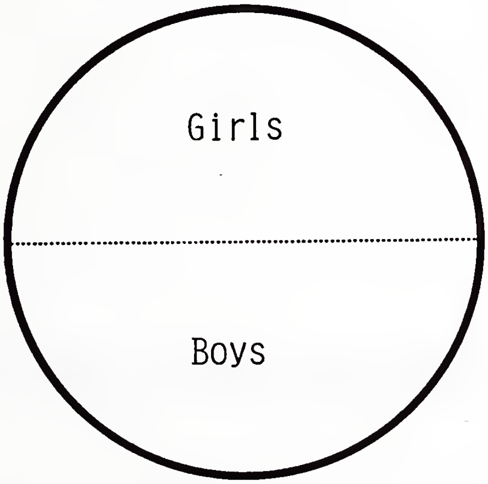
Finally, go back and repeat the above procedure for the errors (wrong turns) data.

Imagine that you are a mouse. You live in a house that is a few blocks away from a cheese factory. You are very hungry and you love cheese, so you decide to try and find your way to the factory. There are many alleys between you and the factory but only one path will get you there. Many alleys lead to dead ends. Start at the circle. This is your home. See if you can find your way to the factory, which is represented by a square. Try to get there as quickly as you can. There are many cats in the alleys, and they are hungry, too.

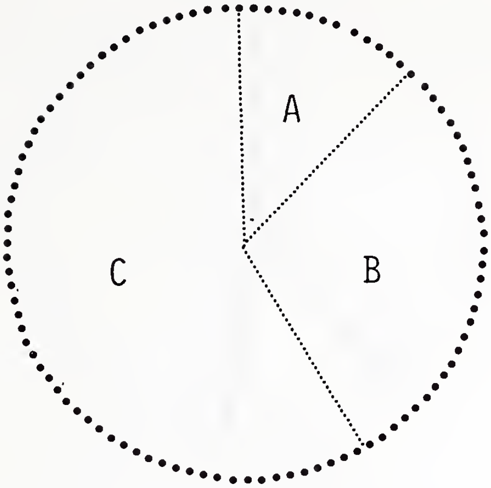

mouse
house
cheese
factory

Circle graphs are another kind of graph that can be used to present information. For example, suppose we wanted to show that half the children in a classroom were boys and the other half were girls. We could draw a circle and divide it in half and label one side boys and the other side girls. Examine the following circle graph. Trace the outer edge of the circle graph completely around. Also trace the horizontal line that divides the circle in half and read the words in the circle graph. [p.104]

Students in a Classroom

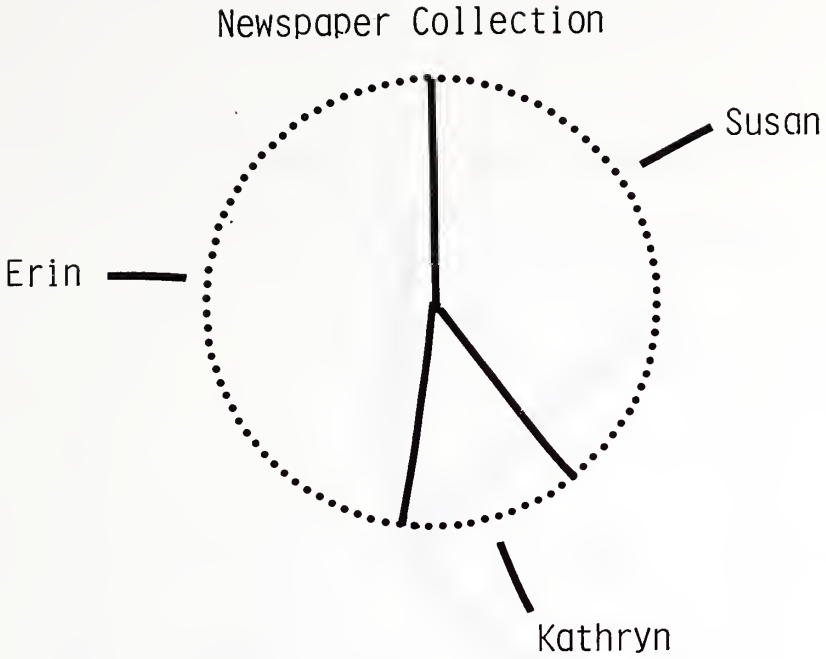


Let's try another circle graph example. Suppose there were three teams of children (A, B, and C) who collected money for the school picnic. Using the following circle graph, decide which team collected the most money and which team collected the least money. [p.105]


Team Collections

A circle graph is sometimes called a pie graph. The pie graph above is cut into three pieces. Each piece is a different size in this pie graph.

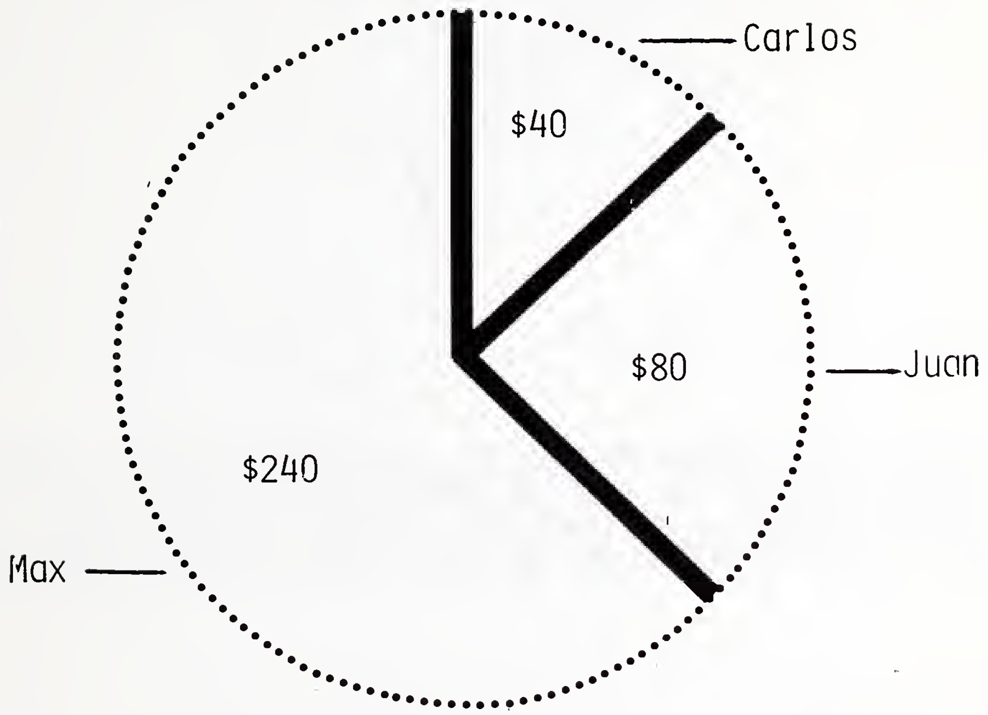
Sometimes in making circle graphs it is not possible to put all [p.106] the written information within the parts of the circle. In those cases the information is written outside the circle and a line is used to point to the part of the circle that it represents. For example, Erin, Kathryn, and Susan had a contest to see who could collect the most newspapers. Read the graph below and decide who collected the most newspapers and who collected the least newspapers.



Sometimes in making circle graphs part of the information is put inside the circle graph and some of it outside the circle graph. The circle graph below shows how much money each of three children collected for the school newspaper. Read the following graph and answer these questions: [p.107]

1. What were the names of the three children?
2. How much money did each of them collect?
3. Who collected the most money?
4. How much money did they collect altogether?

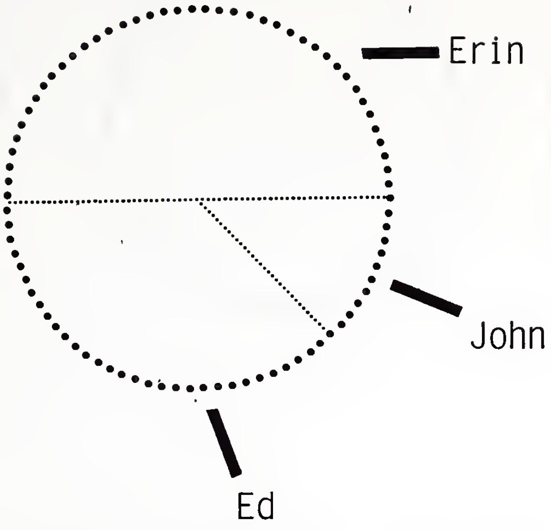
[p.108]


Money Collected for School Newspaper

A contest was conducted by the National Jelly Bean Society to see who could eat the most jelly beans. The contest was narrowed down to three contestants--John, Erin, and Ed. Using the following circle graph, find out who ate the most jelly beans and who ate the least.

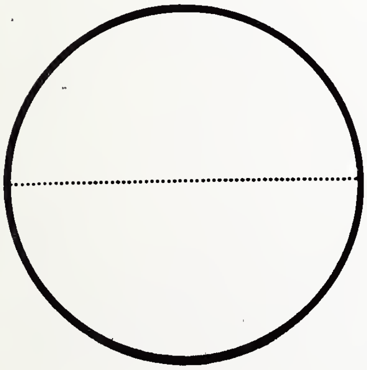
[p.109]

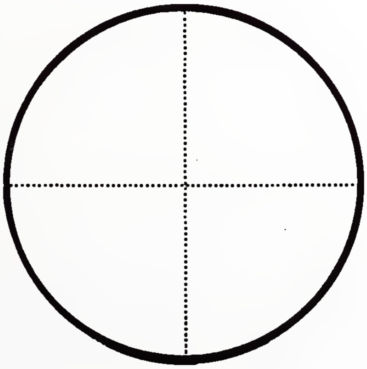
Jelly Bean Contest

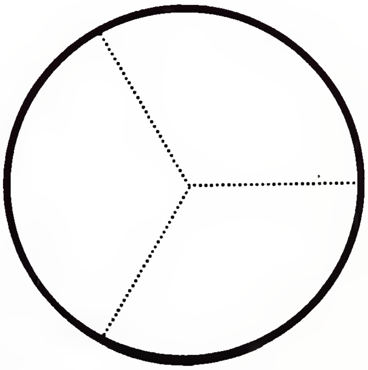


Circle graphs can be divided into different numbers of parts. There are three circle graphs shown below. One is divided into halves, one into thirds, and one into fourths. Examine each graph. Which one shows halves? Which one shows thirds? Which one shows fourths?

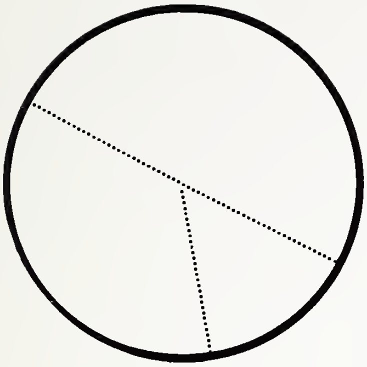
[p.110]

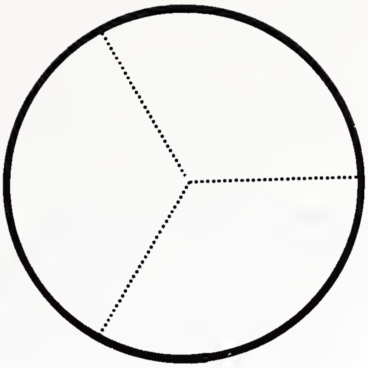

A

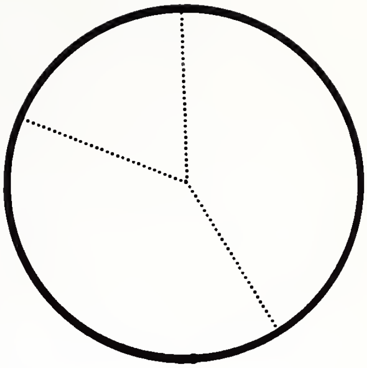

B


C

Hilda receives $3.00/week allowance. She spends $1.00 for school, $1.00 for candy, and saves $1.00. Which of the following graphs would best show how Hilda uses her money?


A

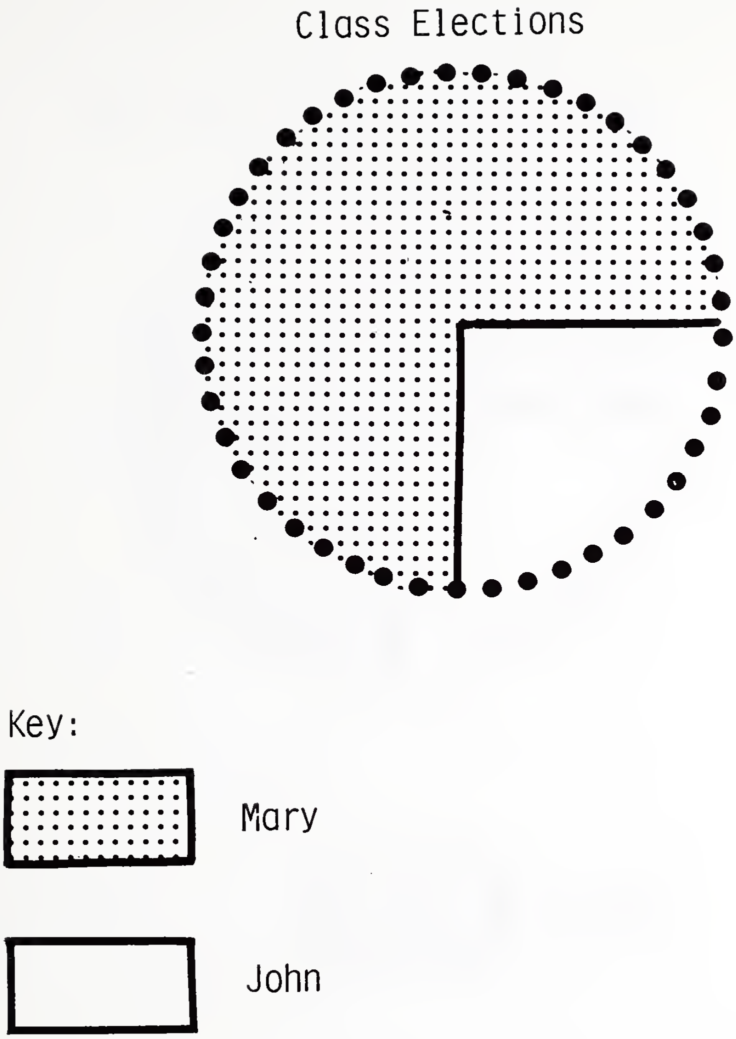

B


C

Circle graphs sometimes have different textures to represent different things. The following graph shows the results of the election for class president. Use the circle graph and the key to answer the following questions: [p.111]

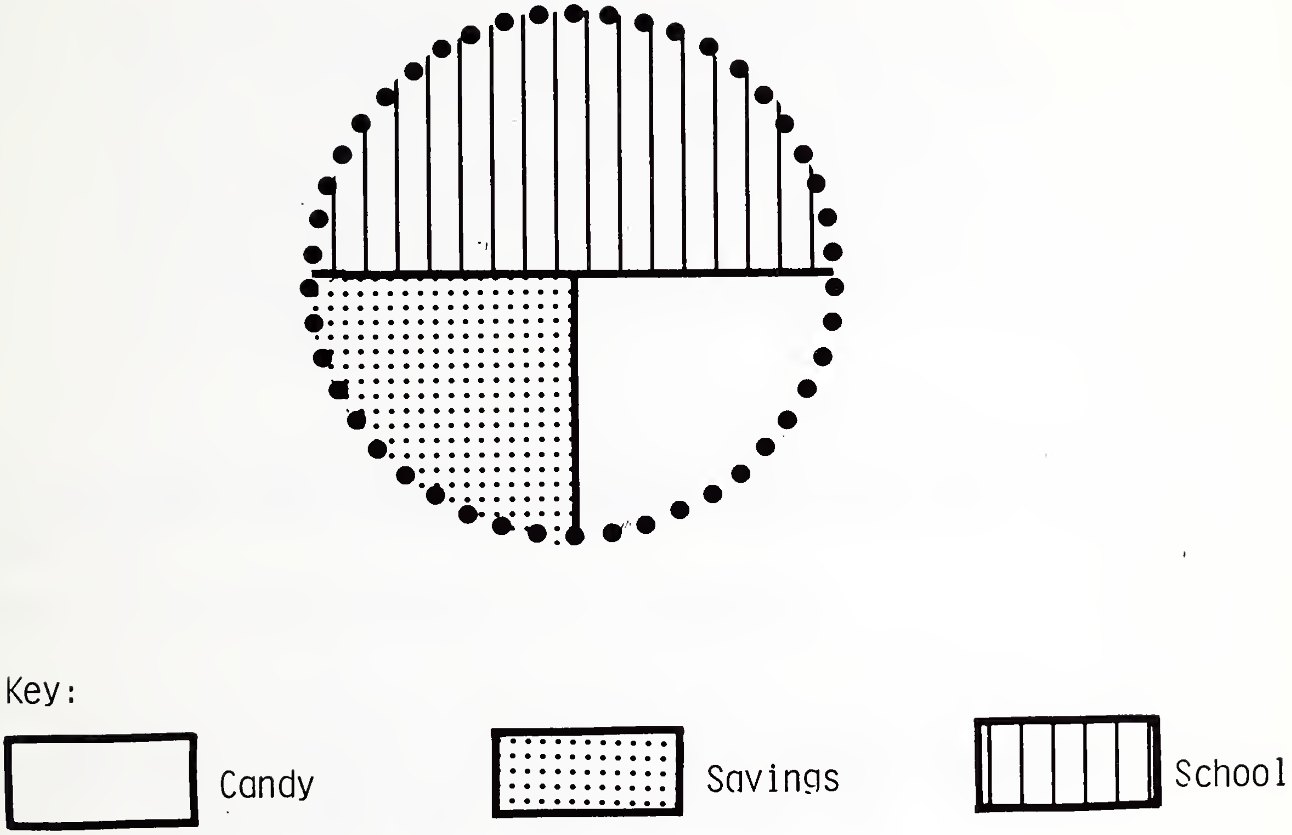
1. Who won the election?
2. Did the winner receive 3/4 of the votes or 2/3 of the vote?

[p.112]



Leroy receives $4.00 per week for allowance. The following circle graph shows how Leroy uses his money. He uses some for school, some for candy, and some he saves. Examine the graph. How much does he use for school, candy, savings? [p.113]

How Leroy Spends His Allowance



TEACHER NOTE

Participation Exercise: Using the circle graph paper, with 12 subdivisions, have the student make a circle graph of this information. To do this, the student must: (1) place a pin at the center of the circle and at the 12 o'clock position on the circle's circumference, and then stretch a

rubber band between them, (2) place a pin at the appropriate position along the circumference of the circle for the first category (in this case, "dogs"--pin located at 6 o'clock), and then stretch a rubber band between it and the pin in the center, (3) continue in like manner for the remaining categories. (Make sure the student understands that the line marking the end of one section is the beginning of the next), (4) glue or simply place braille labels in the appropriate sections of the circle graph.

Participation Exercise: Have the student make a circle graph for information of his or her own choosing. For example, the number of friends of different ages, the number of A's, B's, and C's received last year, the amount of time spent each week engaging in different activities. (circle graph paper with 60 subdivisions is also provided with the program. To determine the pin placement for a particular category (ex. A's) using this graph paper, divide the category value by the total and multiply this fraction by 60. This will give you the number of subdivisions needed for that category.)

Twelve students were asked to name their favorite kind of pet. Six liked dogs best, three liked cats, two liked frogs, and one liked fish. [p.114]

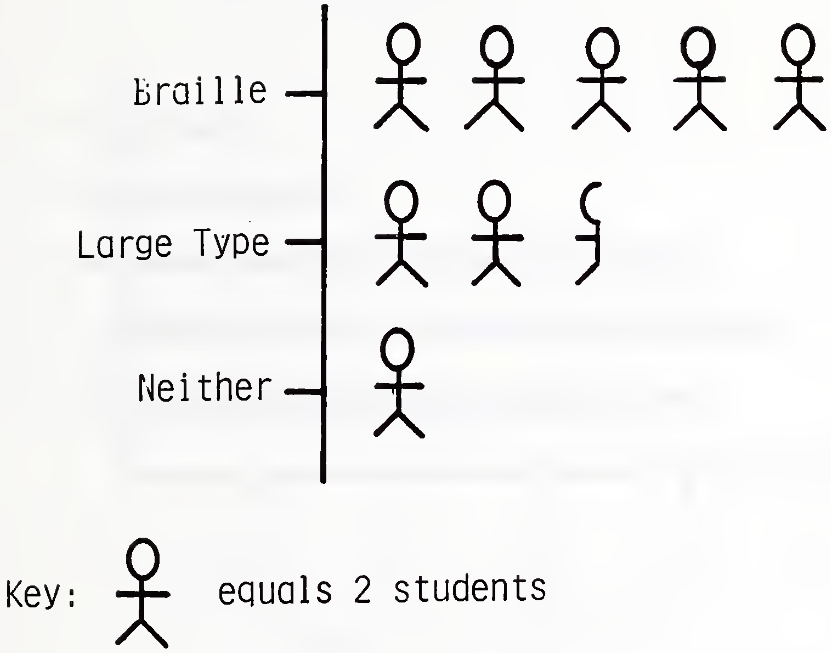
Make a circle graph showing this information.

You have come a long way since you first started this program. Now you should be able to interpret four different types of graphs. Can you name them? The four types are:

pictographs
bar graphs
line graphs
circle or pie graphs

Before we explain a few more things about graphs, let's go over an example of each of the four types to make sure you have them firmly in mind. [p.115] Volume 3

Let's start with a pictograph. The following pictograph tells you how many students in a 9th grade class read braille, large type, or neither. Be sure to examine the key to find out what each picture or symbol represents.



What We Read

How many students read braille? How many read large type? How many read neither braille nor large type? [p.116]

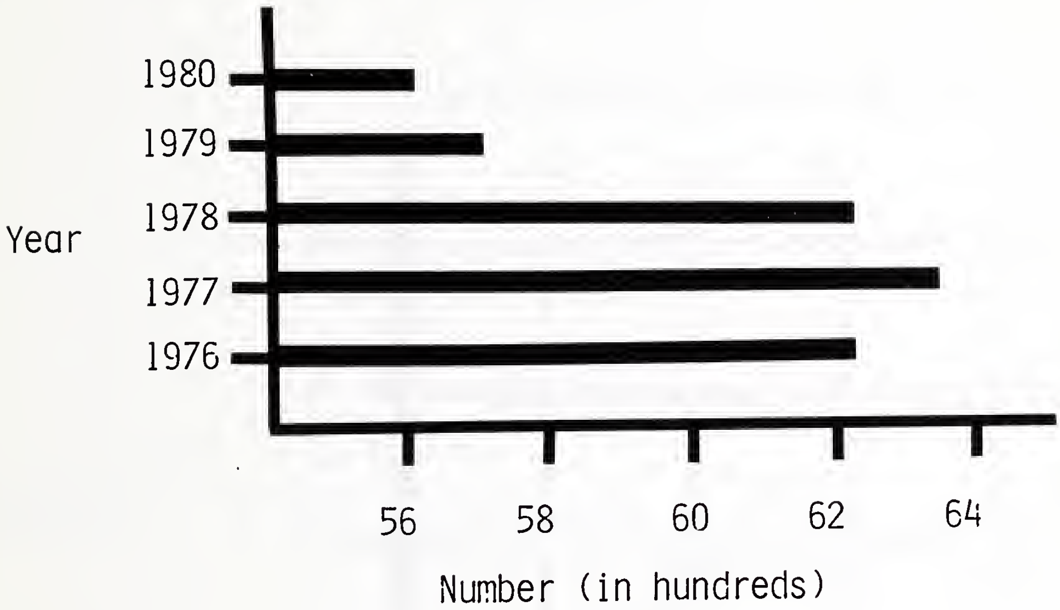
The following bar graph shows the number of blind students in the United States who read braille. Examine the graph and answer the following questions:

1. How many braille readers were there in 1976?
2. In what two years were there 6200 braille readers?

3. In what year were there the least braille readers?
4. Between what two years was there the largest drop in braille readers?
5. What time period is covered in this graph?

[p.117]

Braille Readers



The following line graph shows the approximate number of blind children in the United States from 1976 to 1980. Examine the graph and answer the following questions:

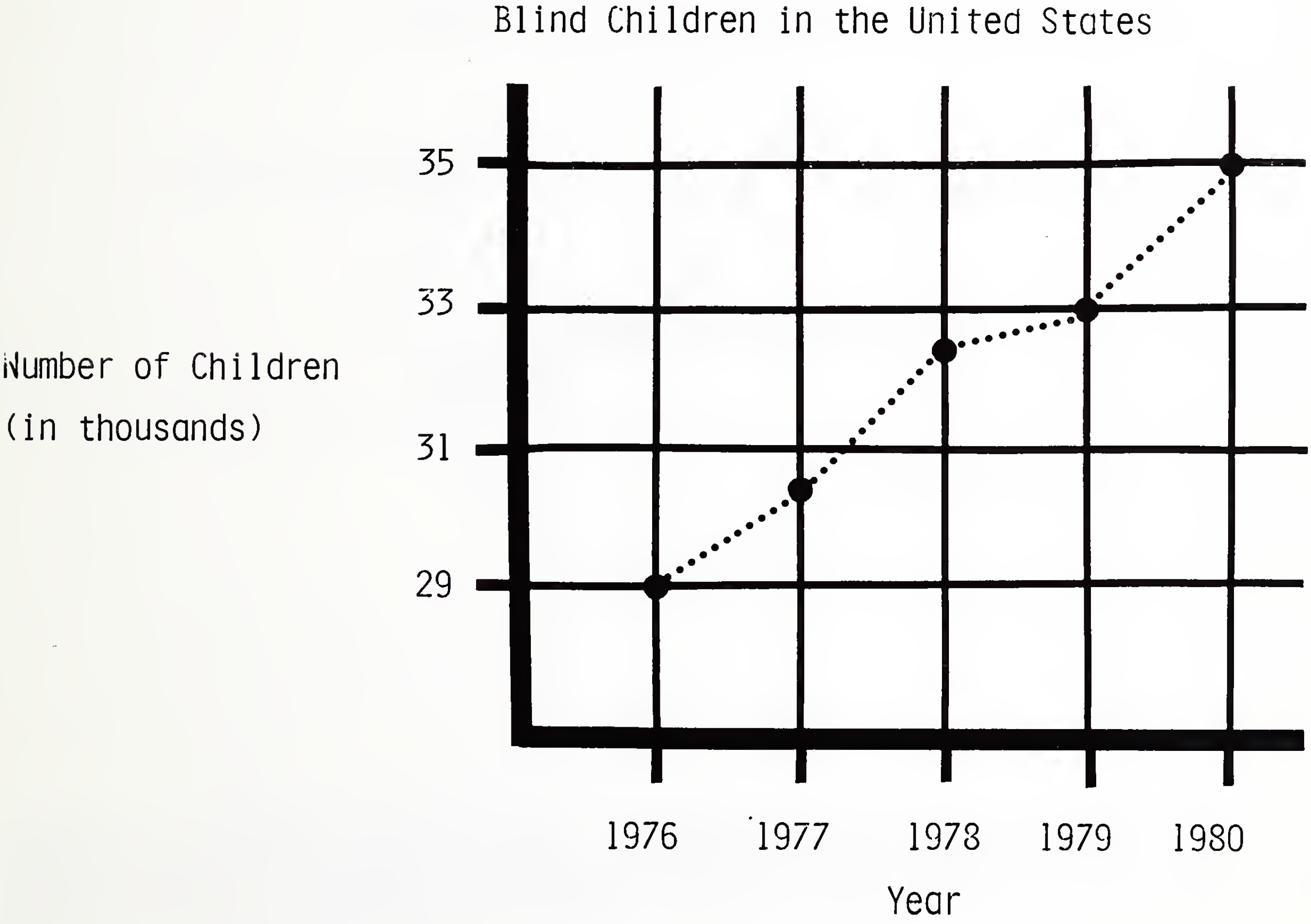
1. What is the label on the y-axis?
2. In what year were there the least number of blind children in the United States [p.118]
3. Approximately how many blind children were there in the United States in 1977?

4. In what year were there 35,000 blind children in the United States?

5. Since 1976, has the number of blind children been increasing, decreasing, or remaining the same?

6. Would you expect more, less or the same number of blind children in 1981?

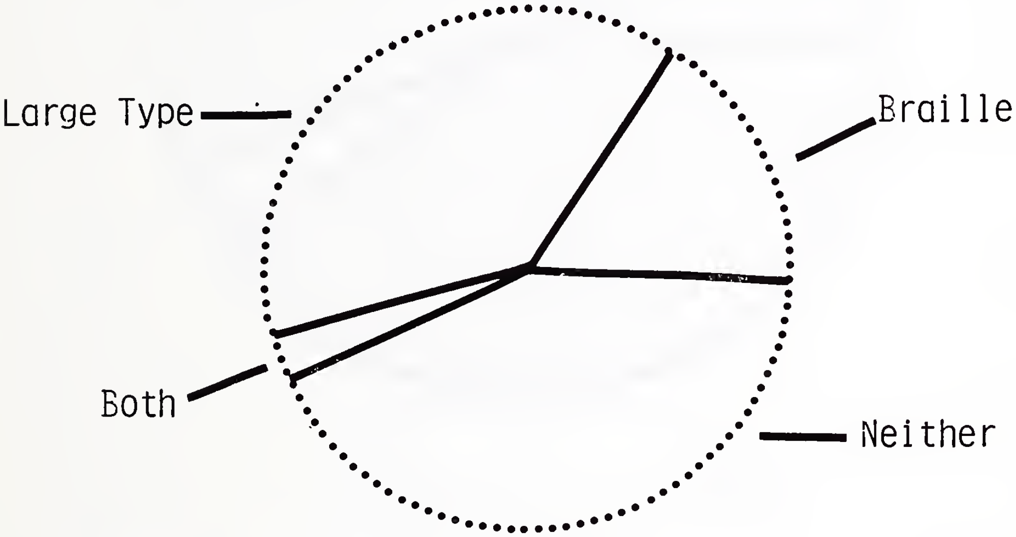
[p.119]



The following circle graph shows the proportions of students in 1980 who read braille, large type, both large type and braille, and neither. Examine the graph and answer the following questions:

1. Which are the largest two categories of readers? [p.120]
2. Which is the smallest category?
3. Are there more or less large type readers than braille readers?

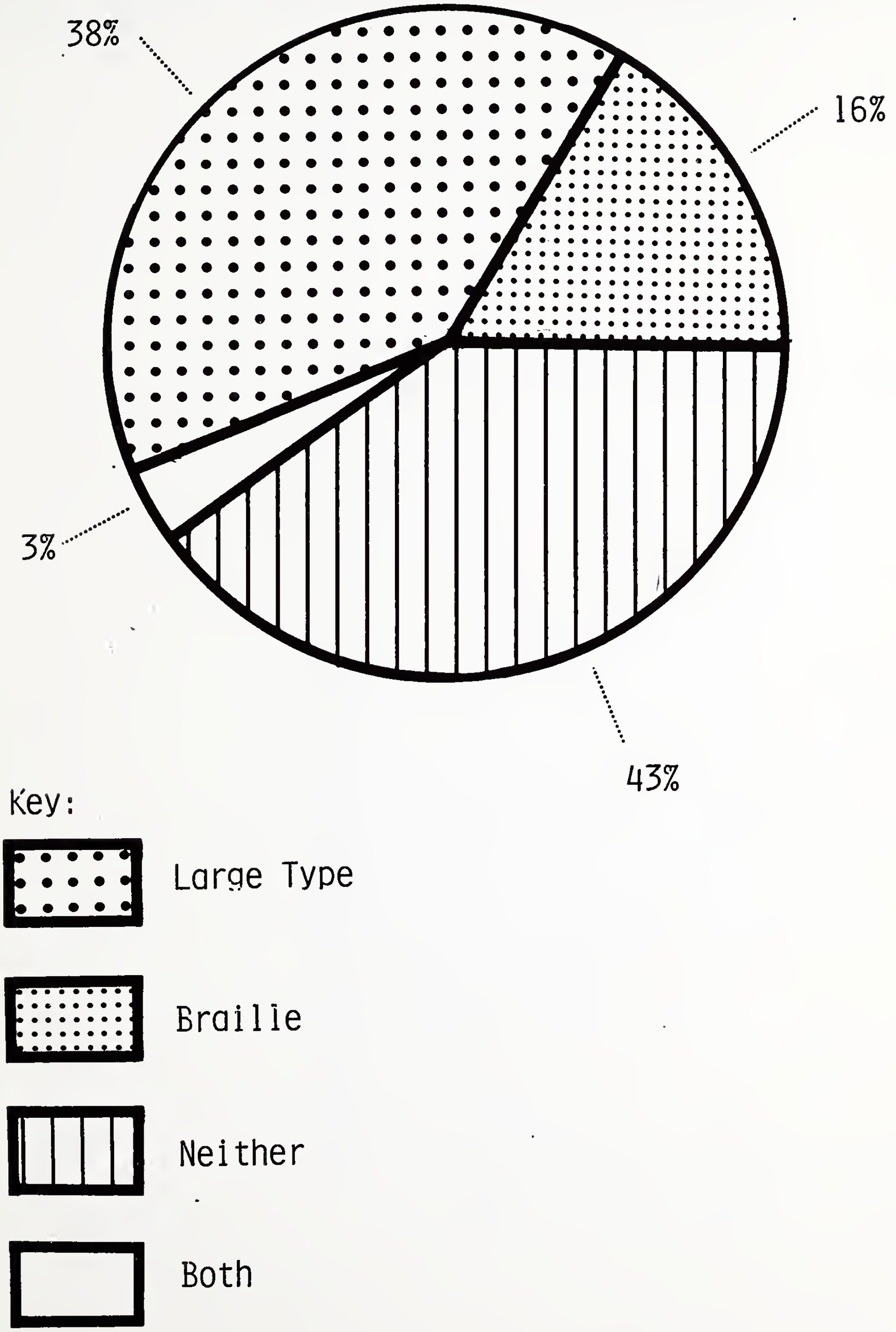
Types of Readers in 1980



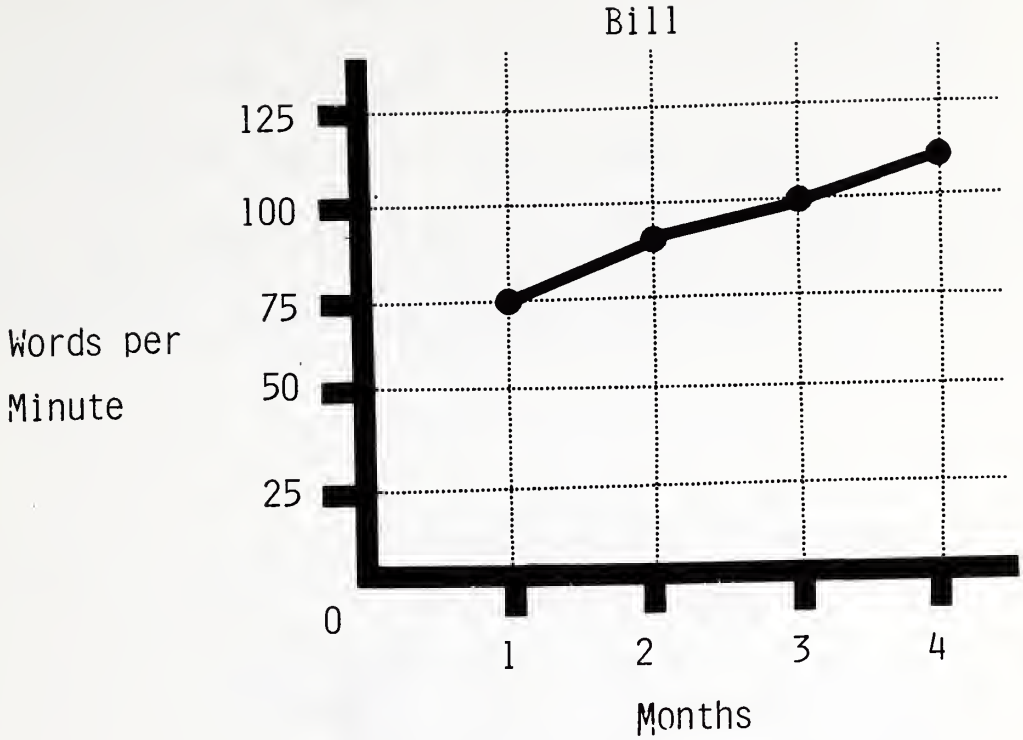
Here is one more circle graph. It shows the percentage of blind students in the United States in 1980 who read braille, large type, both, or neither. Examine the graph and the key and answer the following questions: [p.121]

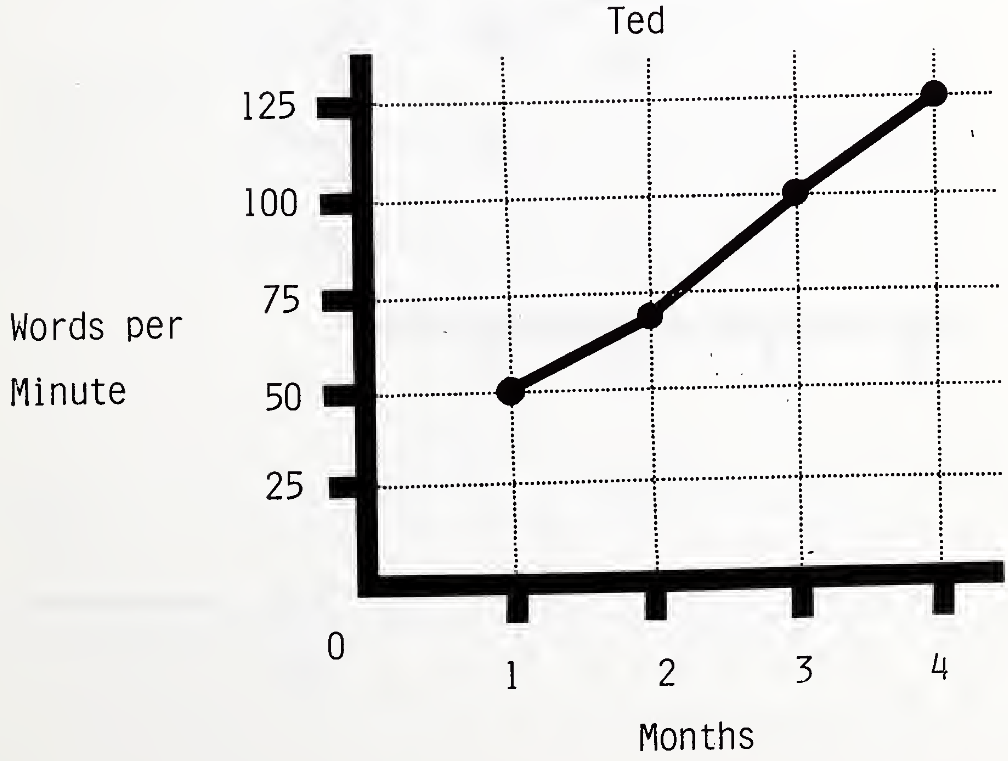
1. What percentage read braille?
2. What percentage read large type?
3. What percentage read both?
4. What percentage read neither?

Percentage of Blind Children Reading Braille, Large Type, Both, or Neither in 1980



Up to this point you have only examined line graphs that have one line. This will not always be the case. Line graphs can have two lines, three lines, or even more. Look at the following two line graphs. Each graph has only one line. The graph at the top shows how a reading program helped Bill increase his reading speed over four months. The graph at the bottom shows how the program helped Ted. [p.123]

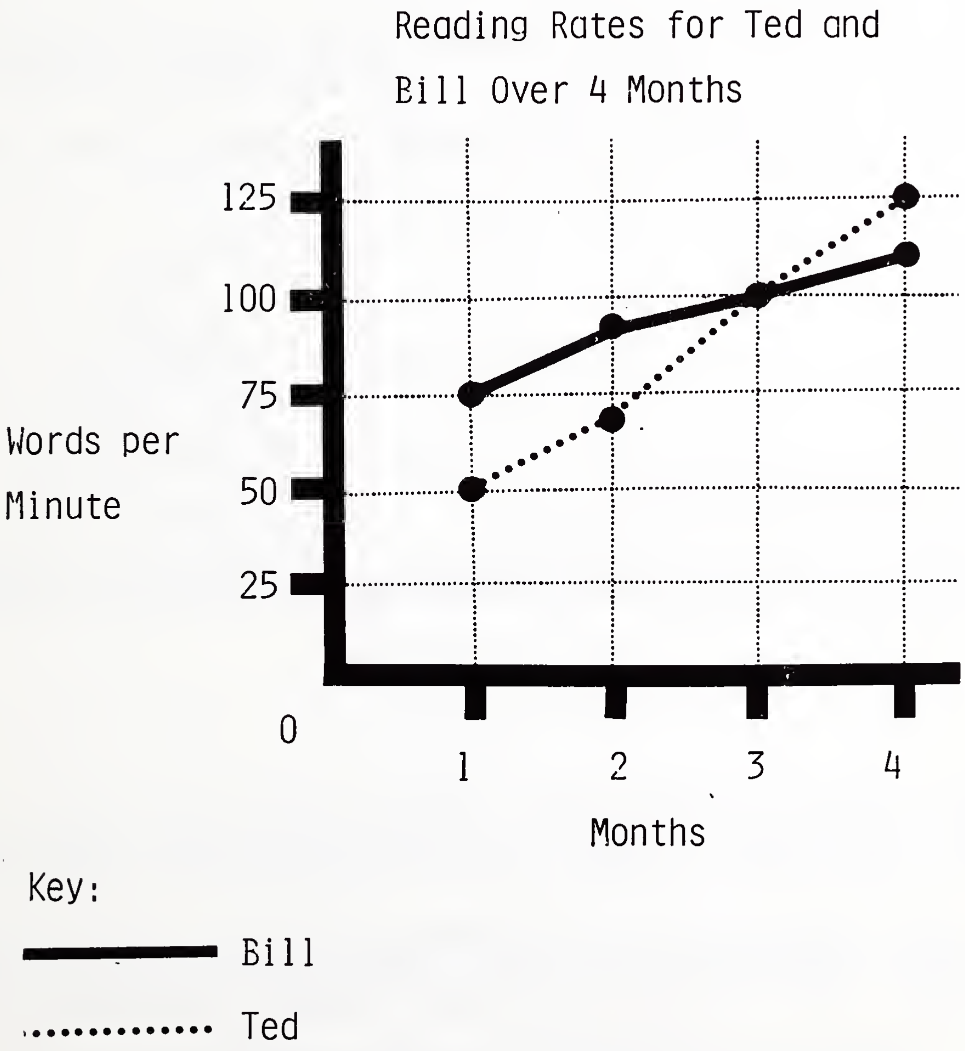

Bill
Words per Minute
125
100
75
50
25
0
1
2
3
4
Months


Ted
Words per Minute
125
100
75
50
25
0
1
2
3
4
Months

Using the two preceeding graphs, answer these questions: [p.125]

1. Who could read faster in the first two months?
2. Did Bill read faster than Ted over the entire four months of the program?
3. Who improved the most over the four months?

In order to answer these questions, you have to go back and forth between the two graphs. This can be difficult and confusing. It would be easier to compare Bill's and Ted's reading speeds if [p.126] both lines were put in the same graph, like this:



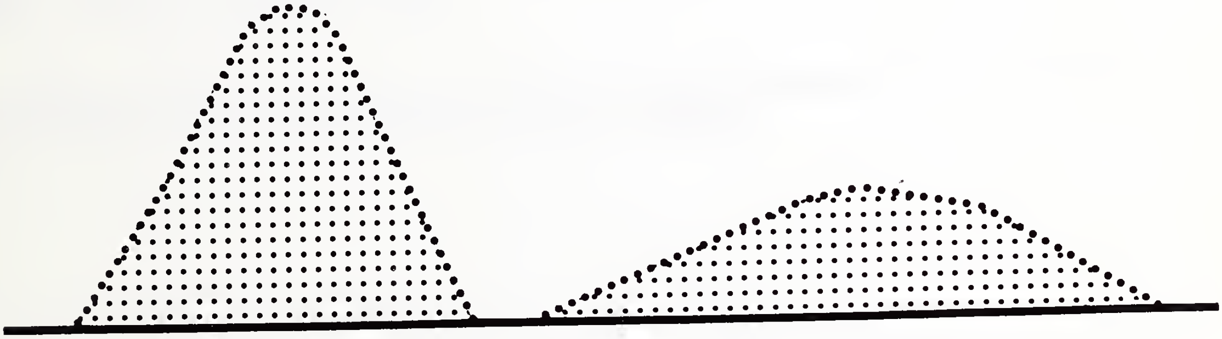
Now you should be able to answer the questions much more easily. By comparing the two lines in the graph you can quickly tell that Bill's line is above Ted's line for the first two months. So Bill was able to read more words per minute than Ted for the first two months. [p.127]

Now look what happened at month 3. The lines cross or intersect. So at month 3, both Bill and Ted were able to read the same number of words per minute. And at month 4, Ted's line is above Bill's line. So at the end of the program Ted could read faster than Bill.

Now trace both lines with your fingertips. You should notice that Ted's line rises more than Bill's line. It is steeper. This tells you that Ted's improvement was greater than Bill's over the four months.

You are probably familiar with the word slope. Slope is used to describe how steep something is.

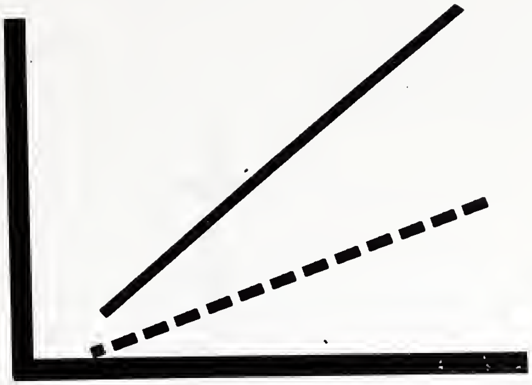
Compare the two hills below. What do you notice about them? [p.128]



The hill on the left is much steeper than the hill on the right. It would be much harder to walk up the hill on the left. The slope of this hill is greater than the slope of the other hill.

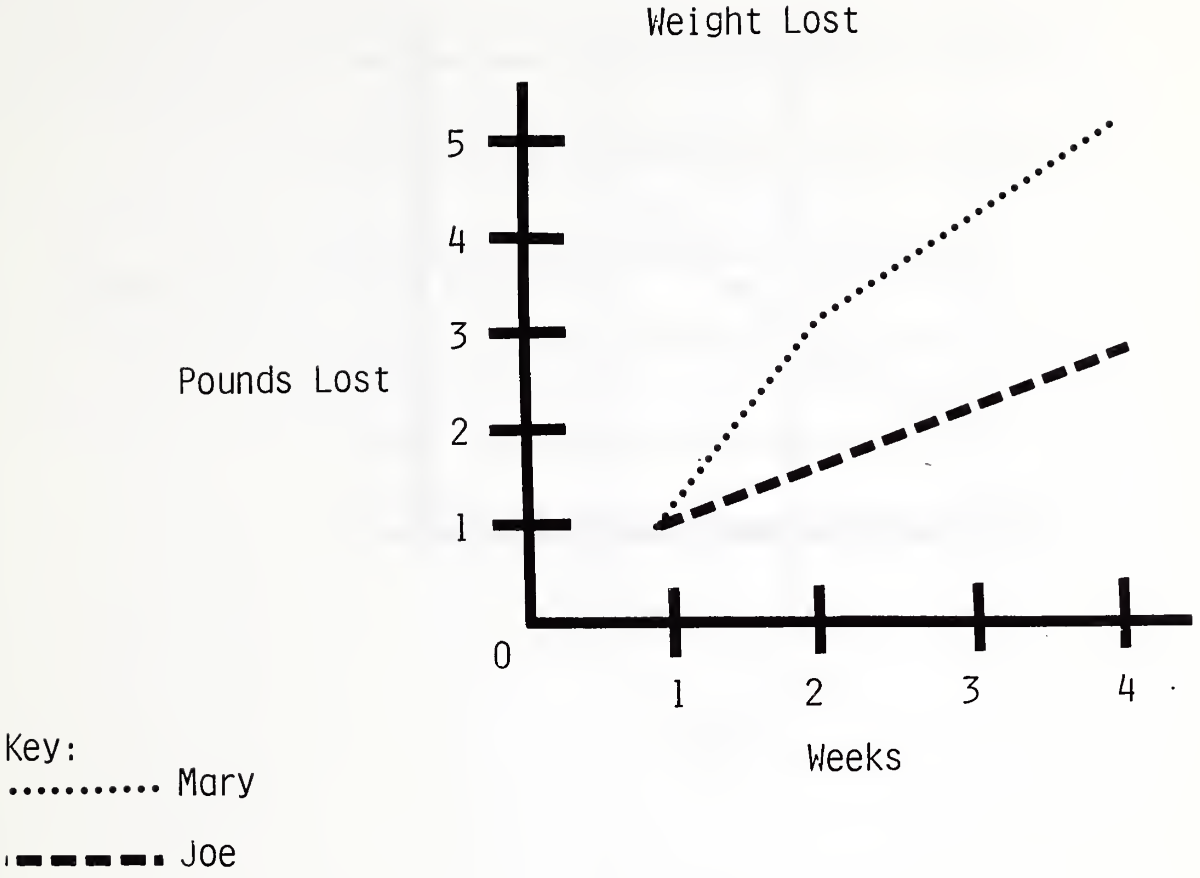
Think about the floor of your classroom. Is it sloped? No. The floor does not go up or down, so it does not have a slope. A stairway does have a slope. So do the roofs of most houses.

We also use the word slope when we talk about line graphs. Examine the following graph. Notice that there are two lines on the graph. [p.129]



Both of the lines have an increasing trend. In other words, they get higher as they go from left to right across the page. But they are not the same. The solid line rises faster than the dashed line. It is steeper. So the solid line has a greater slope than the dashed line.

Mary and Joe both went on a diet to lose weight. The following graph shows their progress over four weeks.

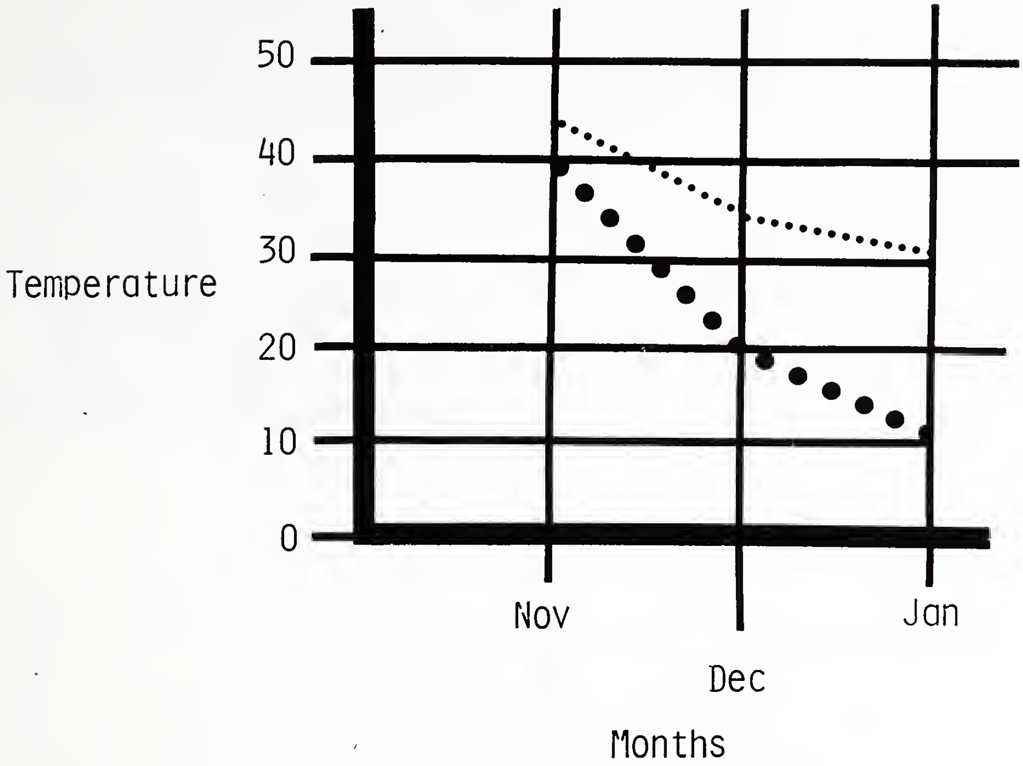


Mary's line is steeper than Joe's line. The slope of her line is greater. This means that Mary was losing weight faster than Joe.

The slope of a line tells you how fast something is increasing or decreasing.

Look at the following graph. It shows the temperature in two cities over three months. [p.131]

Temperature in Two Cities



Notice that the lines show decreasing trends. They fall as they go from left to right across the page. However, the line for city A is steeper. In other words, it has a steeper slope. So the temperature in city A decreases faster than the temperature in city B from November to January. [p.132]

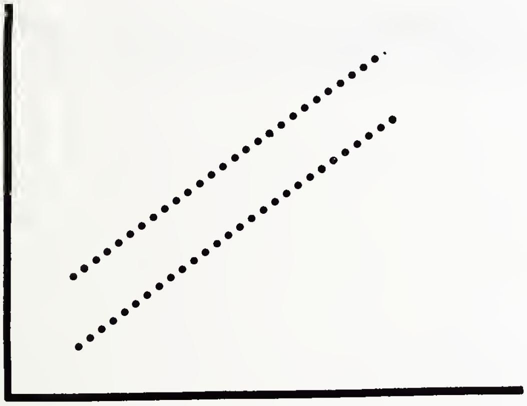
When a line falls, as in the preceding graph, it is said to have a negative slope. So any line showing a decreasing trend has a negative slope.

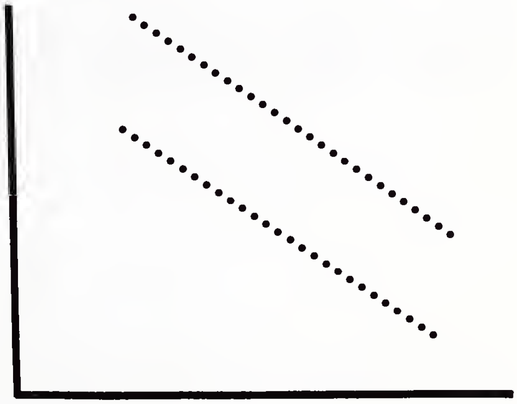
When a line rises, as in the graph on page 130, it is said to have a positive slope. So any line showing an increasing trend has a positive slope.

When one line is steeper than another, it not only has a greater slope, it also covers a greater range of values. In the preceding example, city B's highest temperature over the 3 months is 45 and its lowest temperature is 30. So its temperature ranges from 30 to 45 over the 3 months, a difference of 15 degrees. City A's temperature ranges from 10 to 40 over the 3 months, a difference of 30 degrees. The range of temperatures [p.133] from November to January in city A is thus greater than the range of temperatures from November to January in city B. Can you determine the range of temperatures shown along the y-axis?

When two lines on a graph are parallel to each other, their slopes are the same. Neither line is increasing or decreasing faster than the other line. Look at the following two graphs. The graph on the left shows two parallel lines with increasing trends. They have the same slope. So they are increasing at the same rate. The graph on the right shows two parallel lines with decreasing trends. They also have the same slope. So they are decreasing at the same rate.

[p.134]





The lines on one of the preceding graphs have a negative slope. Which one is it?

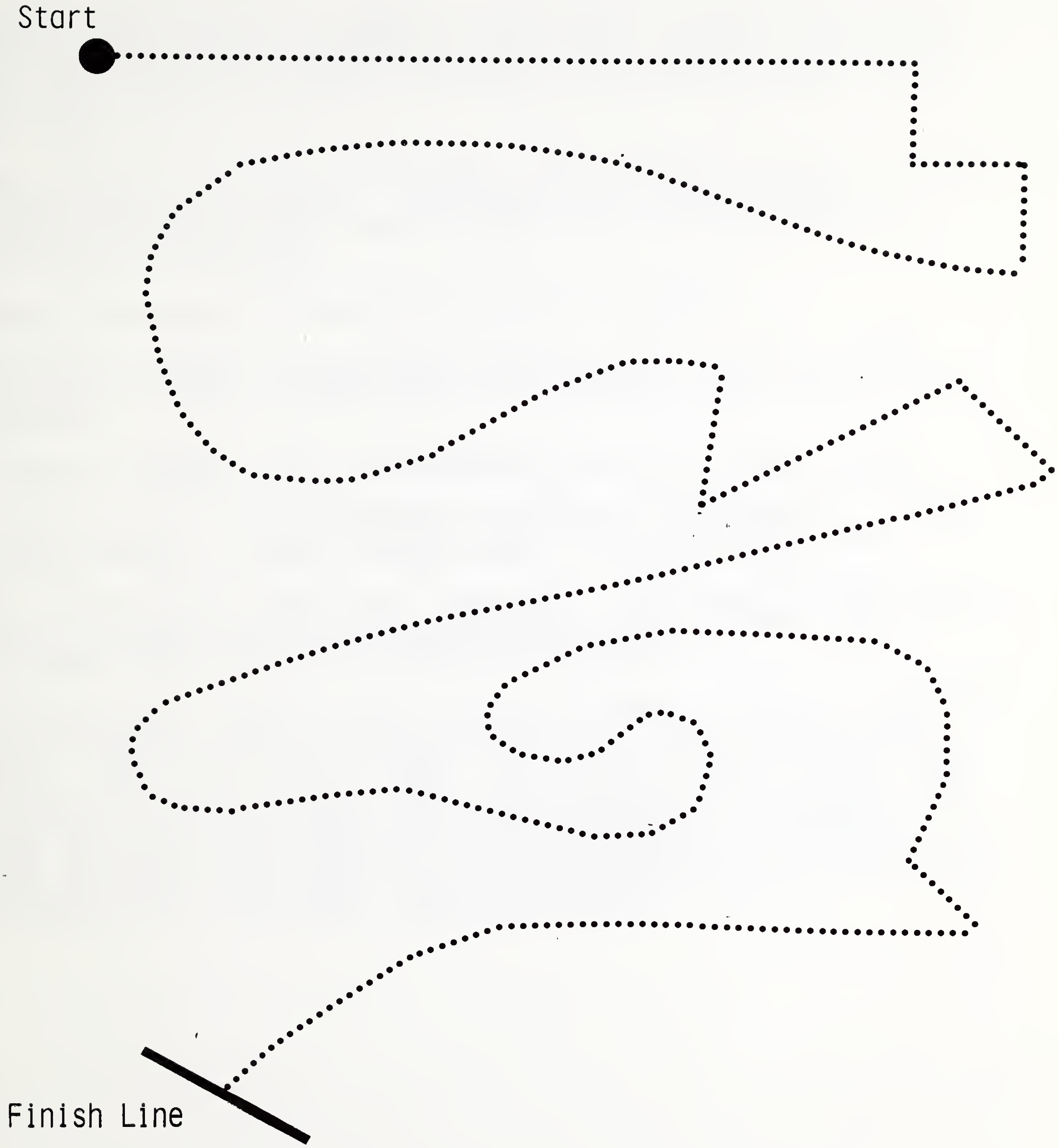
TEACHER NOTE

Participation Exercise: Have the student follow the track four times with the index finger of each hand. Alternate the left and right hands from trial to trial. Time each run with a (stop) watch.

Using the materials provided with the program, have the student construct a double bar graph of the time scores. Ask questions, such as: On which trial was there the greatest (least) difference between your left and right hands? How did your left hand do on the first trial in comparison to the fourth trial?

Next, have the student use the same data to construct a multiple line graph. (Twist the rubber bands used for one of the lines to make it easily distinguishable from the other line). Ask questions, such as: What is the trend of the scores for the right (left) hand? Did the two hands improve at the same rate? Which line has the greater slope? What do you think would happen on a fifth trial?

Your left hand has challenged your right hand to a race. See which one can get to the Finish Line of the race track faster. [p.135]


Start
Finish Line

Growing Pains

[p.136]

Charlie was worried. So many of the girls in his class seemed to tower over him! He began to think that perhaps he might not ever grow any taller than he was now, and since he was just 5'3", and wanted to play on the school basketball team, he was worried.

When Charlie got home from school one afternoon, he went to the kitchen and got an apple. He munched on it as he stood at a window and watched his older brother, Rich, shoot baskets at the hoop on the garage.

"What's the matter, Charlie?" called his mother.

"Was Rich as short as me when he was my age?" Charlie asked. Rich was a junior in high school and nearly a foot taller than Charlie.

"I don't remember, dear, why do you ask?" Mrs. Grove smiled.

"I want to be a good basketball player on our team, but I feel like a midget. I want to know when I will grow some more." [p.137]

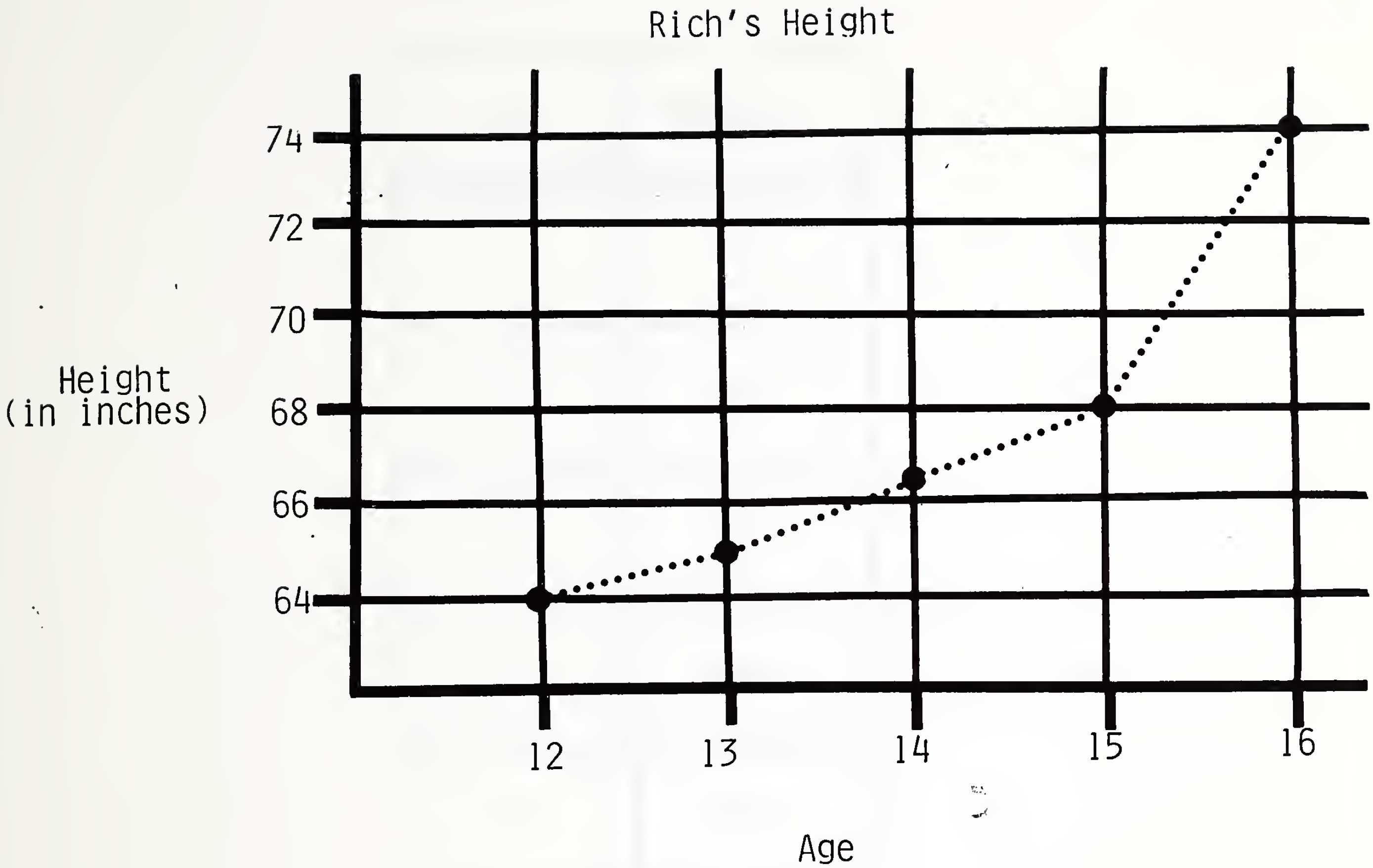
Mrs. Grove said, "Come here, Charlie. I'll show you the place on the doorway where we recorded Rich's height every year on his birthday, and you can see how fast he grew."

On the kitchen doorway was a series of marks. Next to each mark was a number, which was Rich's age at the time the mark was made. There was another number on the other side of each mark showing how much Rich weighed. But there were no numbers telling the feet and inches. So Charlie thought to himself, "I'll have to measure how high each of these marks is and write it down on a piece of paper." He took a yardstick from the closet and did this. His paper looked like this:

Age	Inches
12	64
13	65
14	66.5
15	68
16	74

Now Charlie knew how much Rich had grown, but still he wasn't pleased. "A table of numbers does not tell a story as well as a picture can," he thought. "I could make a picture out of this list that would really show me how Rich grew." And Charlie sat down and made a line graph that looked like this:

[p.139]



This graph told Charlie that Rich had been of just average height up until the time he was around 15 years old. In fact, he grew about one inch every year, which was approximately the rate at which Charlie was growing now. The slope of the line up to age 15 is not very steep, as you can see. But between his fifteenth and sixteenth birthdays, Rich grew six inches. [p.140] There the line on the graph shoots up very steeply. The slope increases.

Charlie looked at this graph with great satisfaction. The graph had shown Rich's growth in an effective way, and now Charlie felt that he, too, had a chance to grow tall enough to play good basketball.

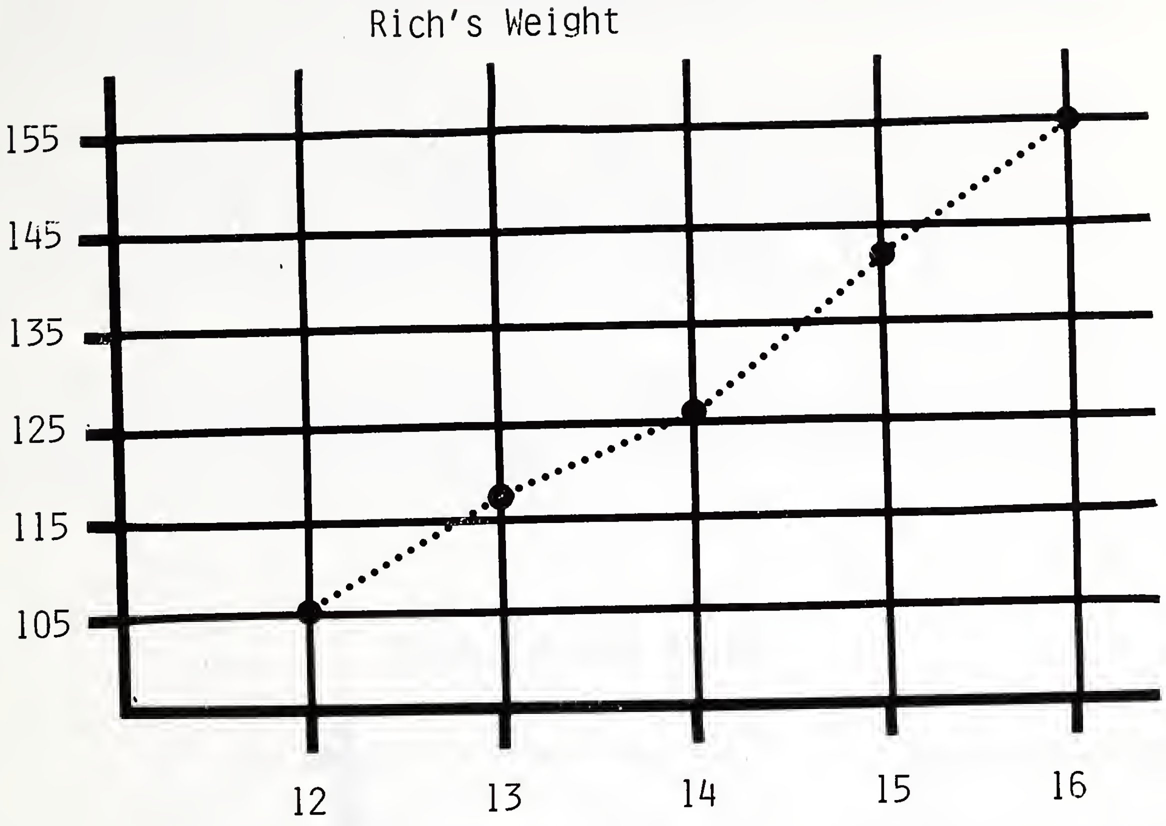
But then Charlie had a sudden thought. He exclaimed to his mother, "Rich is so skinny! I never want to be that thin. I want to play football too, you know." He went to the kitchen doorway once more, to look at the figures showing how much Rich weighed on his birthdays. This is what he saw:

[p.141]

Age	Weight
12	106
13	118
14	127
15	142
16	155

Charlie made a graph of this information, too.

[p.142]



He saw that the slope of the line showing Rich's weight from year to year increased fairly gradually and didn't change very much, even between the ages of 15 and 16, when Rich grew six inches taller.

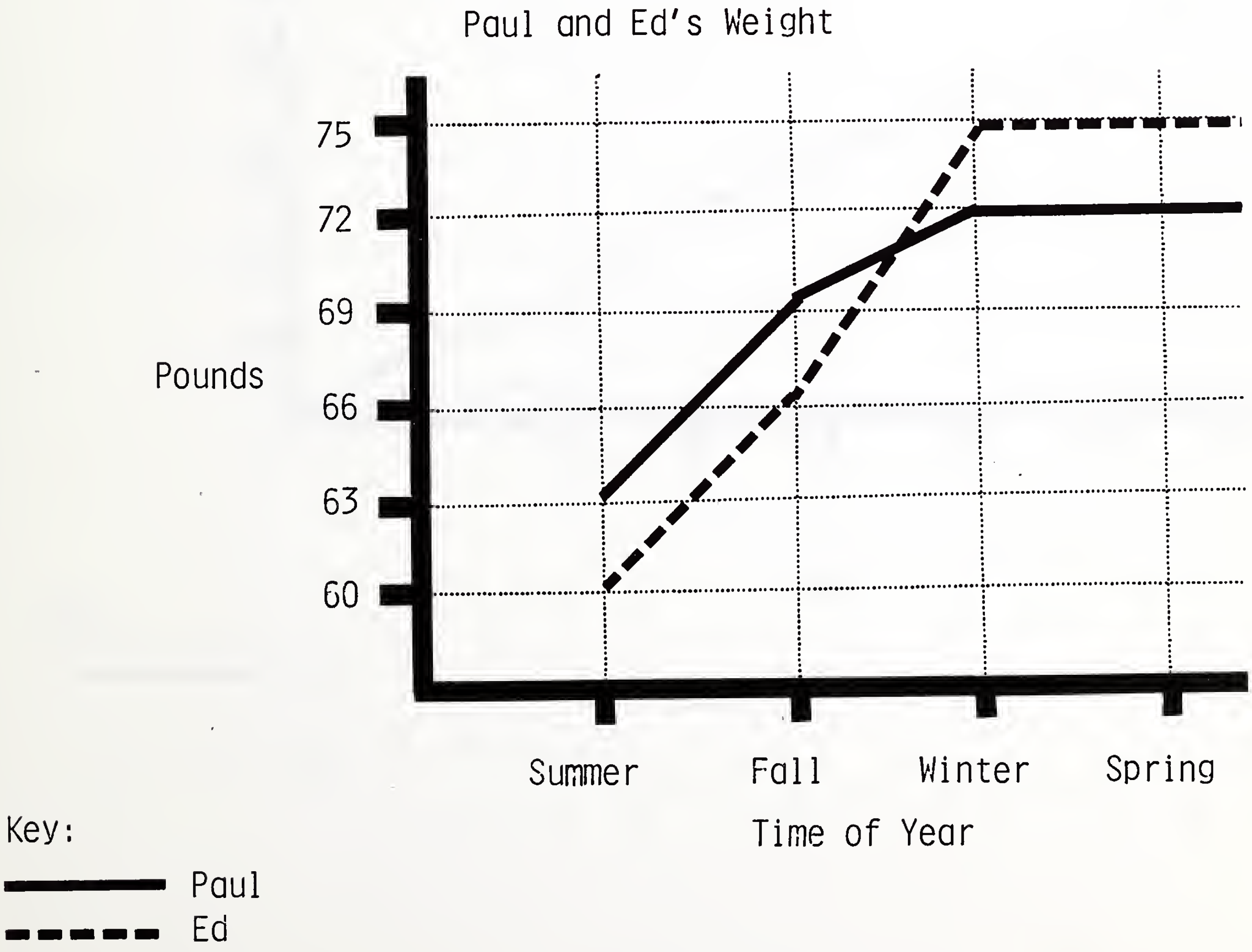
Charlie glanced again at Rich, who was shooting free throws, and groaned. He went straight to the kitchen and grabbed another apple and two bananas. [p.143]

The following graph shows how much Ed and Paul weighed over a year's time. Examine the graph and notice the following: The x-axis shows the time of year and the y-axis shows weight in pounds. Ed is represented by the dashed line and Paul by the solid line. Ed weighed 60 pounds in the Summer and Paul weighed 63. Their lines are parallel from Summer to Fall. So they both gained exactly the same amount of weight. They both gained 6 pounds from Summer to Fall.

In the Fall, Paul weighed 69 pounds and Ed weighed 66 pounds. From Fall to Winter, the slopes of their lines are different. Ed's slope is greater. Therefore he gained weight faster from Fall to Winter than Paul. Paul went from 69 pounds in the Fall to 72 pounds in the Winter, a gain of 3 pounds. Ed went from 66 pounds in the Fall to 75 pounds in the Winter, a gain of 9 pounds. [p.144]

Notice that from Winter to Spring Ed and Paul's lines are both flat; they are both parallel to the x-axis. Can you tell what this means? It means that neither of their weights increased or decreased from Winter to Spring.

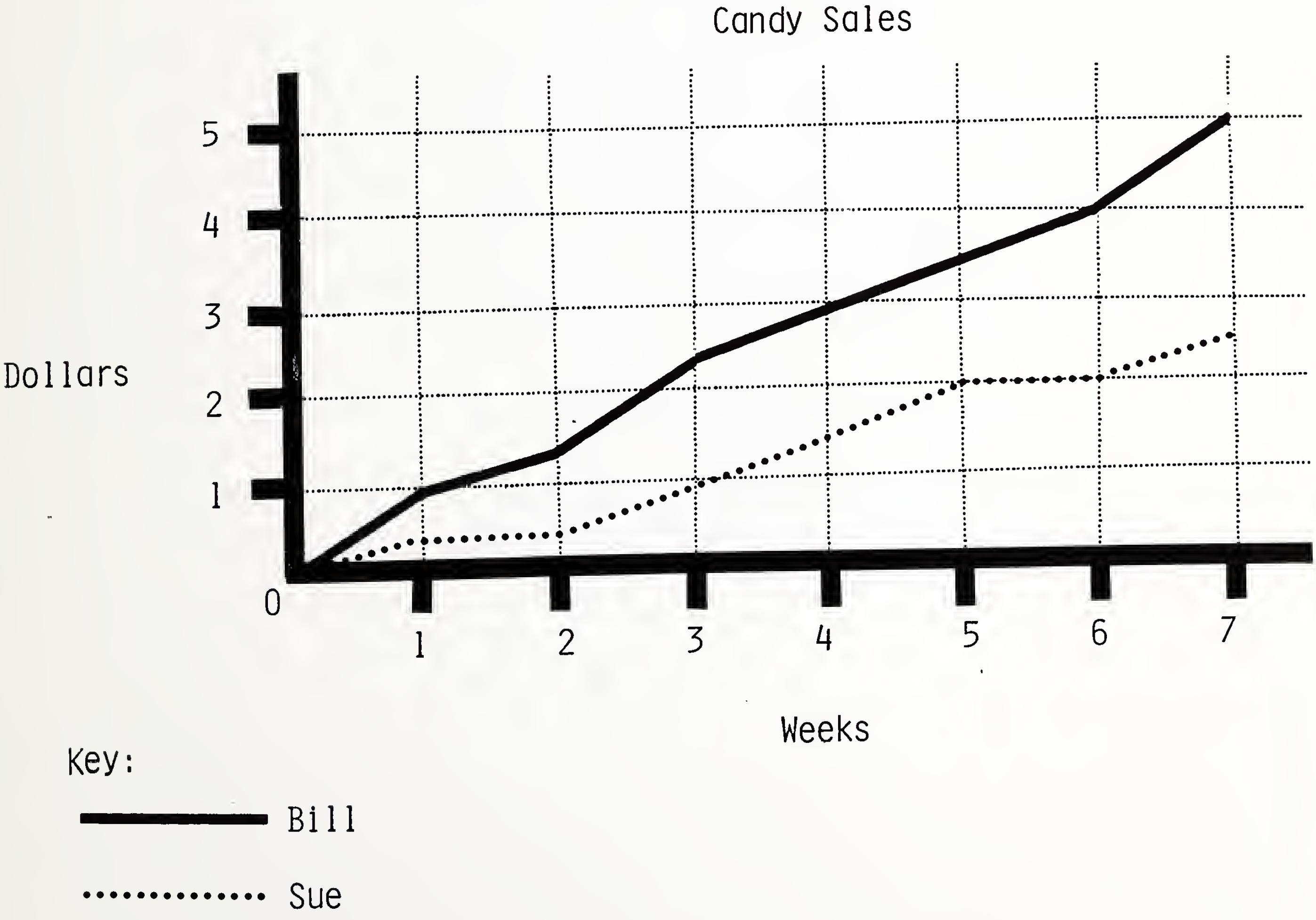
[p.145]



Sue and Bill sold candy they made at home to children in their class. The following graph shows how much money Sue and Bill made over a period of weeks. Examine the graph and answer the following questions:

1. Over how many weeks did they sell their candy? [p.146]
2. Who made money at a faster rate, Sue or Bill?
3. How much money did Bill make at the end of 3 weeks?
4. Who had made $4.00 at the end of 6 weeks?
5. How much more money did Bill have than Sue at the end of 7 weeks?

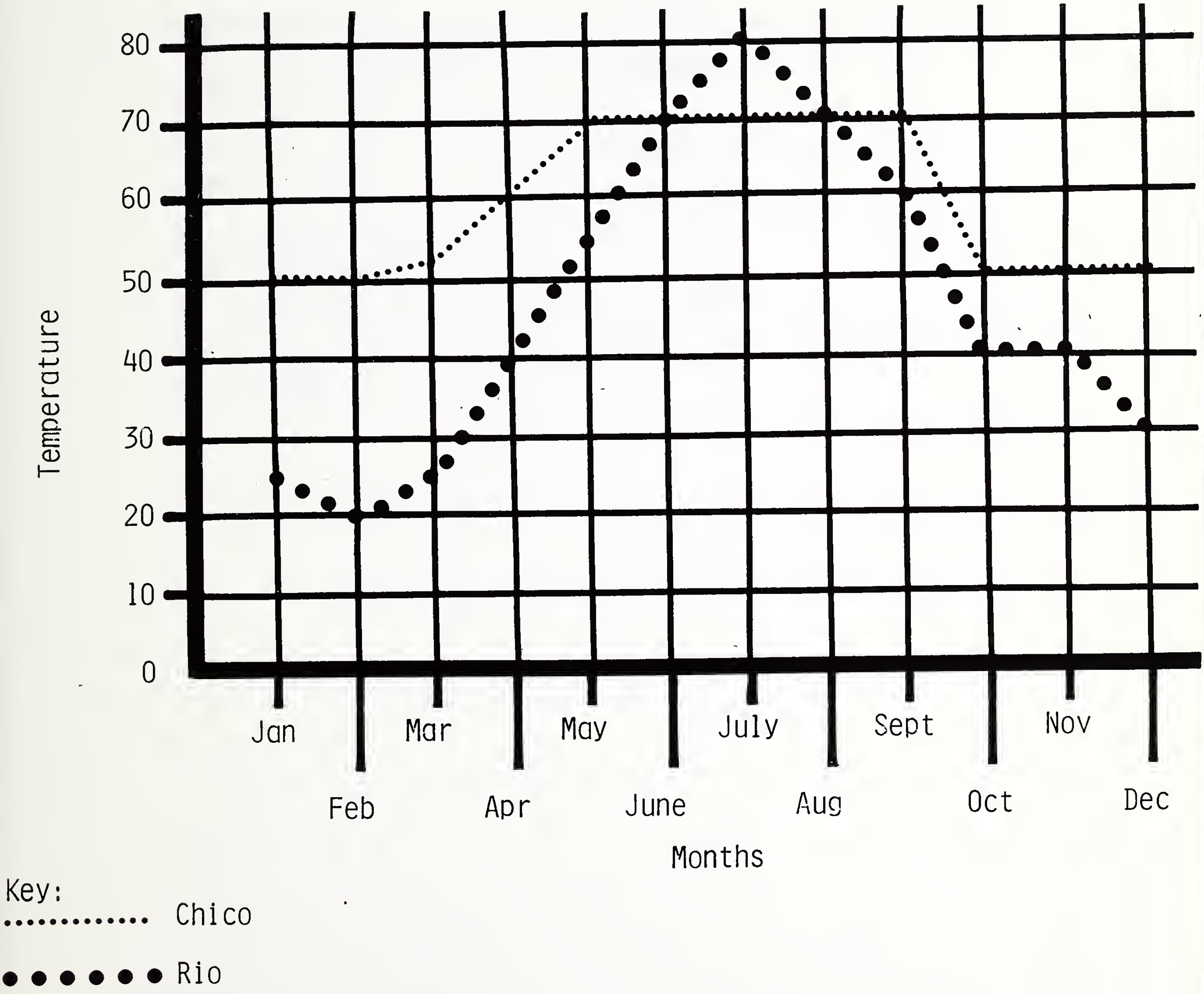
[p.147]



The graph below shows the average monthly temperature over a year's time for two cities in South America. The two cities are Chico and Rio.

1. What is the range of temperature in Rio? In Chico? [p.148]
2. During what months does Chico have its highest temperatures?
3. During what month does Rio have its lowest temperature?
4. During which months do Rio and Chico have the same average temperature?
5. Which city shows the greater variability or change in temperature over the entire year?
6. At what month does Rio's graph line change from a positive to a negative slope?

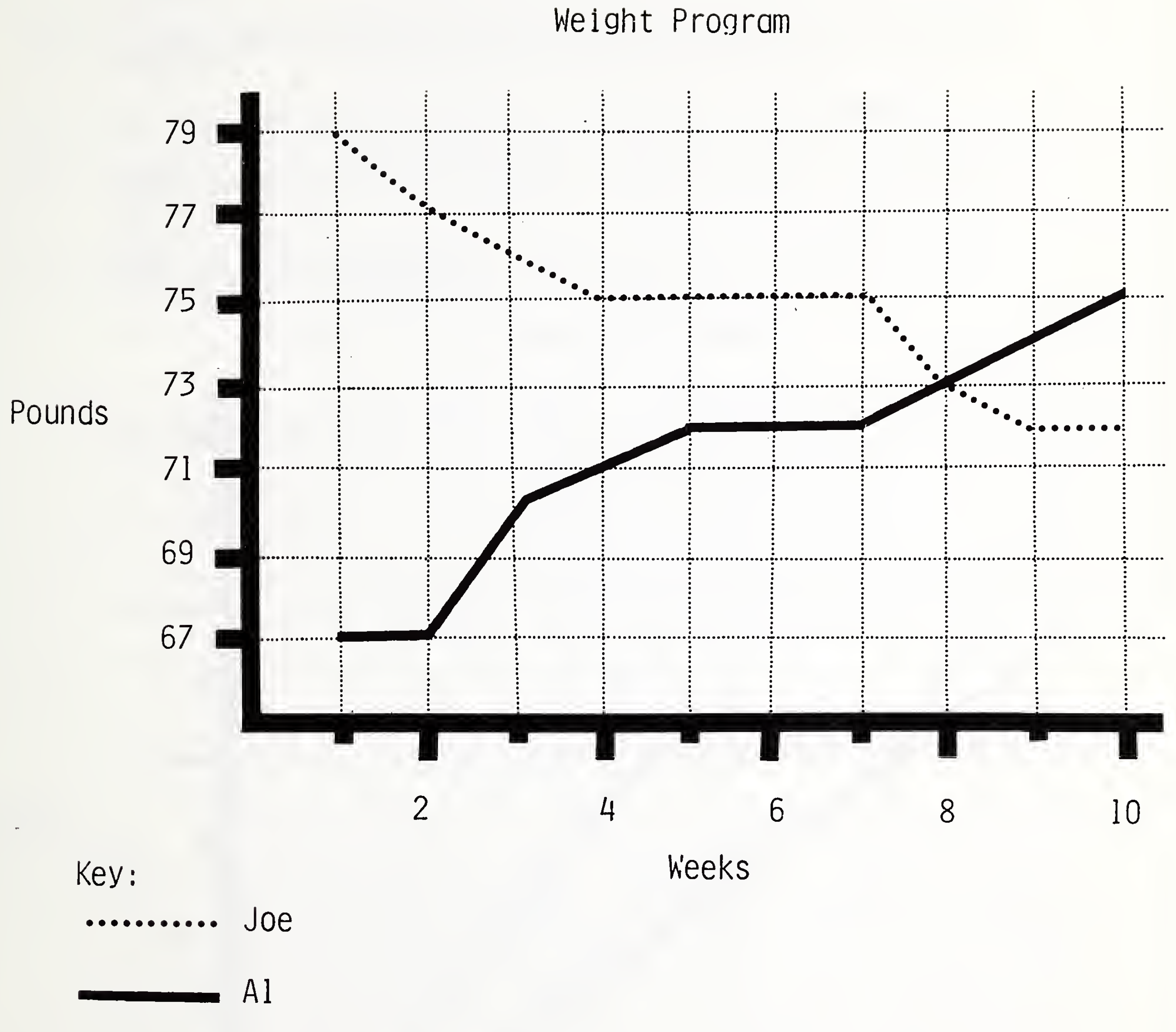
Monthly Temperatures for Chico and Rio



Joe and Al are 6th grade students. Joe wanted to lose weight and Al wanted to gain weight. The graph below shows what happened over a 10 week weight program. Examine the graph and then answer these questions:

1. How much did Al and Joe weigh when they started their program?
2. How much weight did Joe lose over the 10 week weight program?
3. How much weight did Al gain over the 10 week program?
4. How much weight did Joe lose between the 4th and 7th week?
5. How much weight did Al gain between the 2nd and 5th week?
6. At which week were Al and Joe exactly the same weight and what did they weigh?
7. At which week did Al weight 71 pounds?
8. At the end of the 10 week program, who weighed more, Al or Joe?

[p.151]

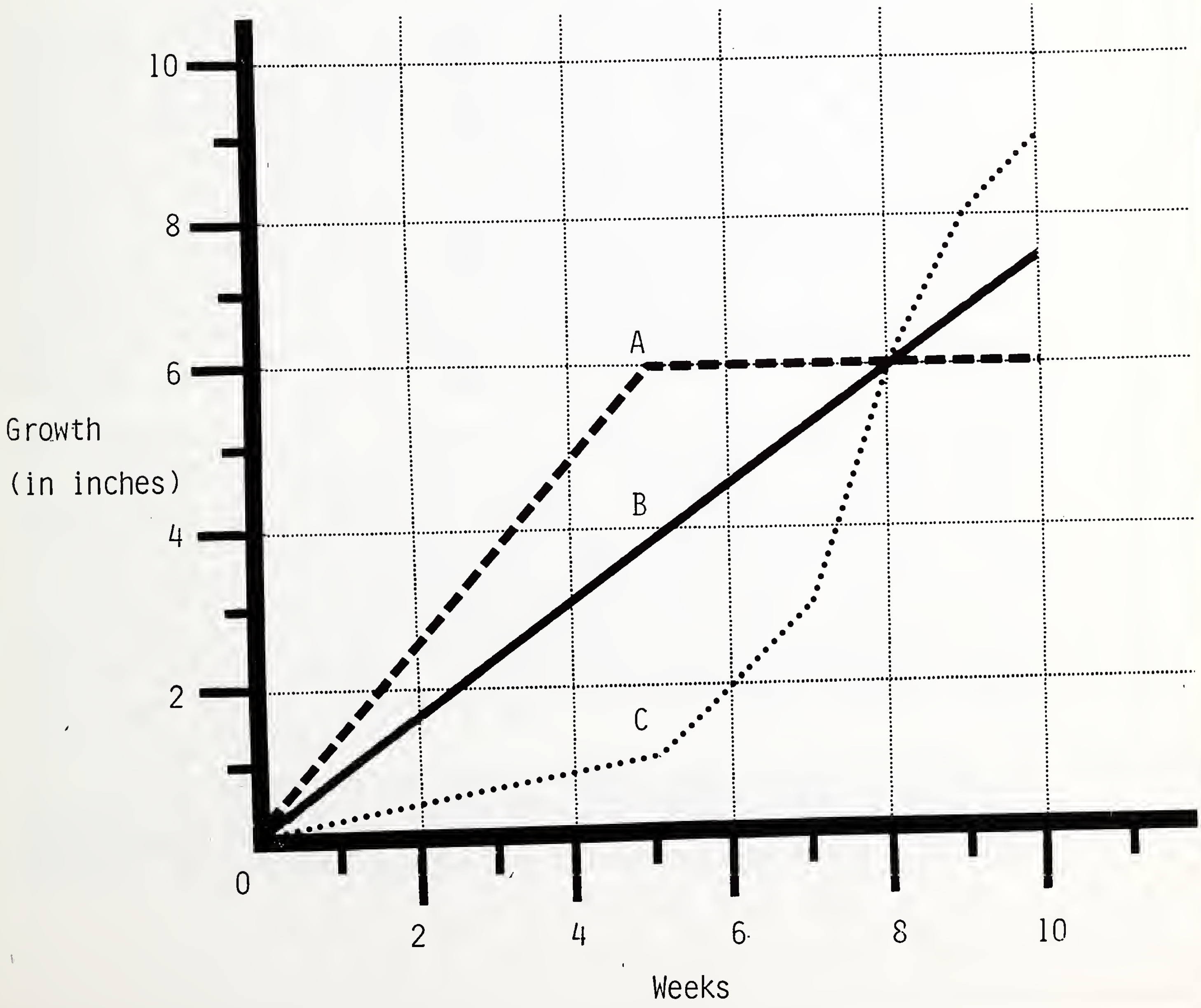


[p.152]

Roberto planted three seeds in three separate flower pots as part of an experiment for his science class. At the end of each week he measured how tall each plant had grown. He labeled the flower pots A, B, and C. Using the graph that Pat made, answer the following questions:

1. How long did the experiment last?
2. Which plant was the tallest at the end of 4 weeks? Which was the shortest?
3. How many weeks did it take for plant C to grow 3 inches?
4. At the end of which week were the plants all the same length?
5. At the end of which week did Plant A stop growing?
6. Which plant grew the fastest from the 6th to the 8th week?
7. How tall do you think plants A, B, and C will be at the end of 11 weeks?

Science Experiment



TEACHER NOTE

To save time, you may want to read this "story" to the student and have him or her refer only to the graphics.

Thump, Thump

[p.154]

The students in a health class at Winding Way Elementary School were discussing physical fitness. Tom said he read in the newspaper that a lot of people in the U.S. are in very poor physical shape. The main reason, he said, is that most of us do not exercise enough. We spend a lot of time sitting in front of the television set. We ride elevators instead of using the stairs. We drive to places in a car when we could walk. As a result, many people are not very healthy.

The class though it would be fun to see for themselves how exercise can improve one's fitness. So they decided to do a little experiment. It was agreed that some of the students would exercise for 15 minutes every day, swimming laps in the school's pool. Some of the other students would swim laps in the pool every day for 30 minutes. The rest of the students would not exercise in the pool at all. By comparing the three groups of students, the class hoped to find out whether exercise did any good. Since there were 30 students in the class, Mary suggested 10 be put in each group. [p.155] They all put their names in a hat and mixed them up. Then ten names were drawn from the hat. These students were put in Group 1, the no exercise group. The next ten drawn were put in Group 2, the 15-minute exercise group. The last ten were put in Group 3, the 30 minute exercise group. They all decided that the experiment would be tried for 3 months.

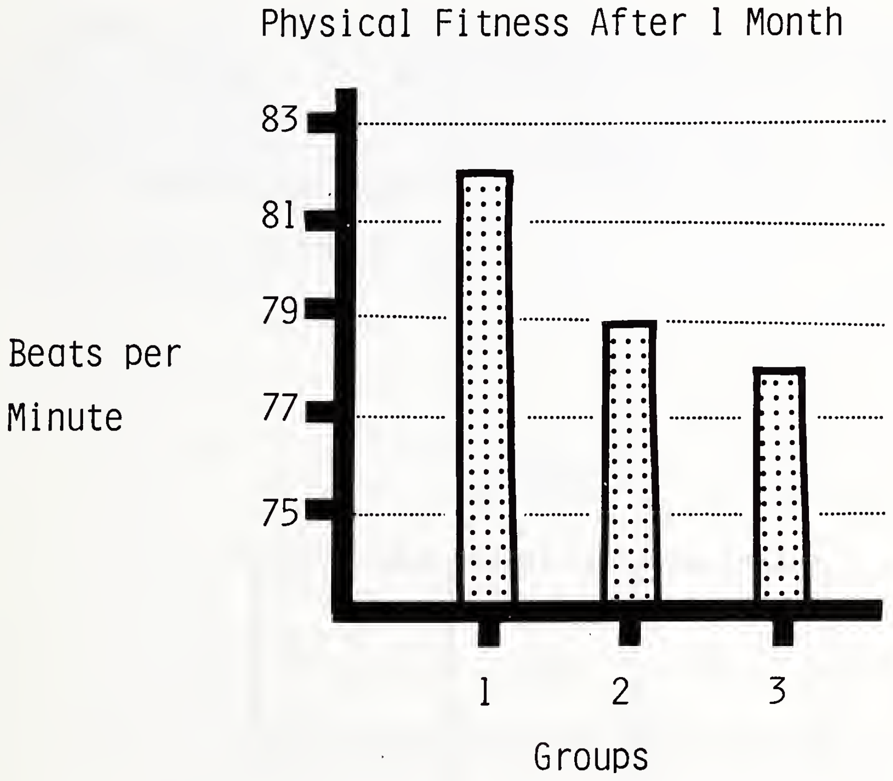
The problem now was to figure out a way to tell how fit a person is. The students had read in their health book that the heart beats slower the healthier you are. The heart is a muscle that pumps blood to all parts of the body. It needs exercise, just like the muscles in the arms and legs. When the heart is not very strong, it has to beat more times each minute to do its job. A strong heart can pump out more blood each time it beats, so it can do the same job with fewer beats each minute. The class decided that this would be a good way of measuring physical fitness in their experiment. So they borrowed a stethoscope from the school nurse. A stethoscope is an instrument doctors use to listen to a person's heart.

The teacher suggested that each student walk up four flights of stairs. [p.156] At the top of the stairs, the teacher would listen to the student's heart and count the number of times it beat in one minute. This would indicate how fit each student was. At the

end of each month of the experiment, the students would walk up the same stairs and have their hearts listened to. By comparing the monthly progress of the three groups, the class could tell whether or not exercise really helped improve physical fitness.

After the first month, the students decided to see how the experiment was going. To do this, they calculated the average number of heart beats of the students in each group and made a bar graph. It looked like this:

[p.157]



The teacher had the class discuss the meaning of the graph. Did it show that the exercise was working? Were the students who exercised for 30 minutes fitter than the students who exercised for 15 minutes? How did these two groups compare with the group that did not do any exercise?

Suzy raised her hand and said: "I think our experiment is working. The bar for group 1 is higher than the bars for groups 2 and 3. This means that the students who have not exercised are not as fit. Their heart rate was higher when they reached the top of the stairs." "Very good, Suzy," said the teacher, "but what about the students who exercised for different amounts of time each day. Are the students who exercised for 30 minutes more fit than the students who exercised for only 15 minutes?" Joe said: "I don't think so. Look at the second and third bars in the graph. They're almost the same height." Mary added, "So that means that 30 minutes of exercise is not much better than 15 minutes." "So far, that's true," said the teacher, "but remember that we still have two months to go before the end of the experiment." [p.158]

At the end of those two months, the students had a lot more data to look at. Teri asked: "How can we make sense out of all these numbers?" Tom had an idea. "Let's organize them in a table," he said. "Since we measured everybody's heart rate for three months, we'll need a table with three columns, one for each month. And we'll need three rows, one for each group." [p.159]

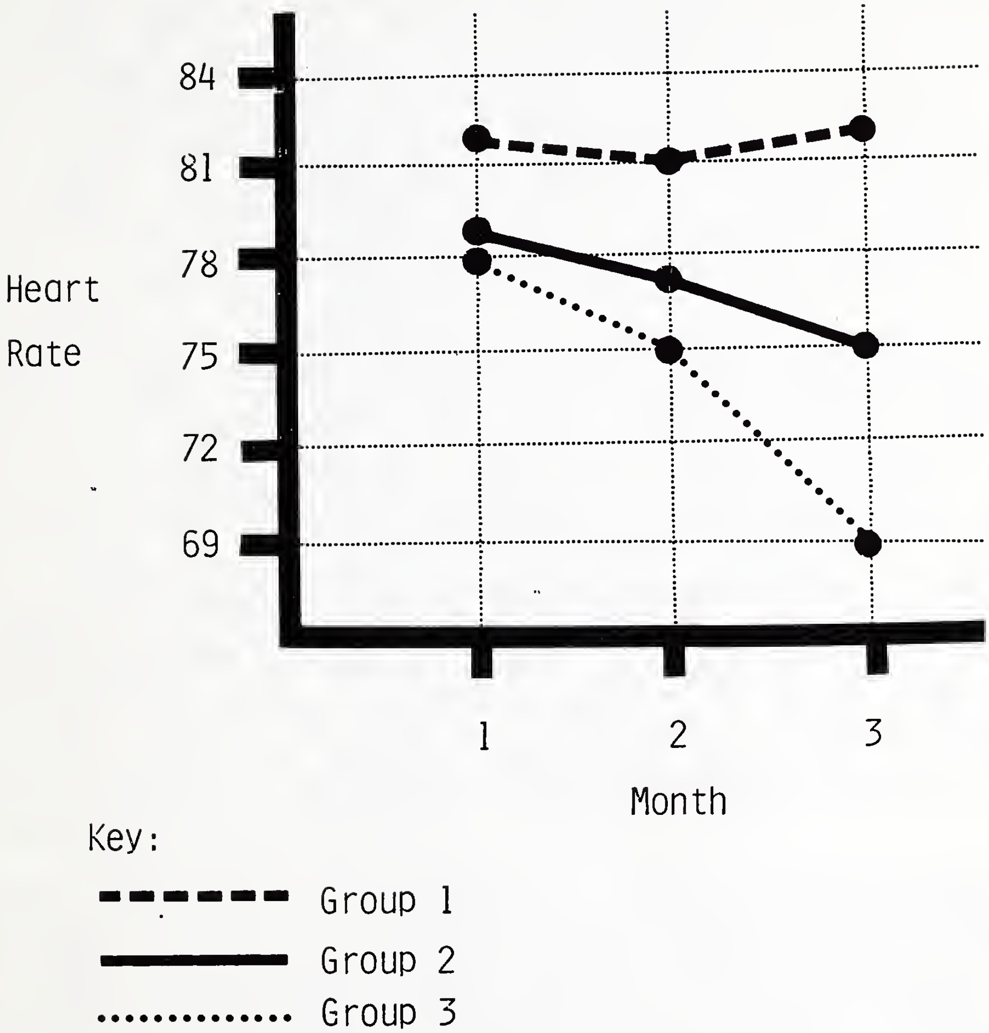
This is what the table looked like:

		Month		
		1	2	3
Groups	1	82	81	82
	2	79	77	75
	3	78	75	69

The teacher then asked, "What does the table tell us about the students in group 3, who exercised every day for 30 minutes?" Joe said: "Well, if you look at all of the numbers in row 3, you can see that the heart rate of that group went down from the first month to the third month of the experiment. It started out at 78 beats a minute and ended up at 69 beats a minute." Mary said: "The students in group 2, who exercised for 15 minutes, [p.160] also improved during the experiment. They went from 79 beats a minute down to 75 beats a minute. But group 1 stayed about the same through the whole experiment." Joe spoke again: "That's right, Mary. And look at column 3 in the table. The three numbers there tell you that the exercise groups finished the experiment in better shape than the group that didn't exercise. The hearts of the no exercise group were beating much faster after they reached the top of the stairs."

Sharon said: "A table really is useful. It helps organize a lot of numbers so that you can make sense out of them. But it's still kind of difficult to look at all of the numbers for each group and then compare them to see what happened in the experiment. Isn't there another way we could organize the data?" "Yes, Sharon, there is," said the teacher. "We can also make a picture out of the numbers to help you understand them better. This kind of picture is called a graph. Let's take the numbers in the table and make a line graph." This is how the data looked when made into a line graph:

Physical Fitness Experiment



The teacher asked, "What do the three different lines represent?" [p.162] Teri said, "That's easy. Each line represents one of the three groups of students. The key tells you which line represents which group. Each line shows how the heart rate of that group changed over the three months of the experiment." "Very good, Teri," replied the teacher. "Now examine each line and see what happened to each group over the three months." "Well," said Joe, "the line at the top of the graph is almost horizontal. This means the heart rate of that group stayed almost the same during the whole experiment. This was the no exercise group. The line for this group is also higher up on the graph than the other two lines. This means that the no exercise group had a higher heart rate than the other two groups through the whole experiment. The line in the middle shows what happened to the 15 minute exercise group. It is more like a diagonal line and falls as it goes from left to right across the page. This means that the heart rate of group 2 decreased over the three months of the experiment. In other words, there is a decreasing trend in heart rate for group 2." Erin spoke up. "Group 3's line at the bottom [p.163] of the graph is similar to the line for group 2. It also shows a decreasing trend in heart rate over the three months of the experiment." "That is true," said the teacher, "but are they different in any way?" "Yes, they are," replied Erin. "The line for group 3 is steeper than the line for group 2. It has a greater slope. This means that the heart rate of group 3 decreased more from month to month than did the heart rate of group 2." "That is only partly true, Erin," said Tom. "Look at the first half of both of the lines and compare them. They are almost parallel to each other. So, from the first month to the second month, the heart rate of group 3 did not decrease much more than the heart rate of group 2. Now look at the last half of both lines. Group 3's line at the bottom is much steeper than Group 2's line. At month 3 the lines are farther apart than they were at month 2. So Group 3 made much more progress than Group 2 during the third month of the experiment."

"Very good, class," said the teacher. "Now you can see how a graph can tell a story. Our line graph showed what happened in [p.164] our experiment. It told us that exercise improves your physical fitness. Both 15 minutes of exercise and 30 minutes of exercise reduced heart rate over the three months of the experiment. The heart rate of the no exercise group did not change. The graph also showed that 30 minutes of exercise reduced heart rate more than 15 minutes of exercise did. And, finally, it showed that the greatest difference between the two exercise groups occurred during the third month of the experiment.

"Well, that is the end of the experiment," said the teacher. "Unfortunately, those of you who were in the no exercise group did not get to benefit from the experiment. So, starting tomorrow, you will get your chance to improve your physical fitness, too. Five laps in the pool each day." Group 1 moaned in unison.

TEACHER NOTE

The following aids are available from the American Printing House for the Blind. All of them are very useful for furthering the student's expertise in tactile graphic interpretation.

Tactile Graphics Kit--a kit of tools, materials, and information for the construction of tactile graphics of all types--graphs, maps, diagrams, and tables. Although intended primarily for teacher use, it is possible for students to construct simple displays with the kit.

Graphic Aid for Mathematics--a kit consisting of an 18" x 19" cork board, flat spring wires, push pins, and rubber bands. The purpose of this aid is to enable students to construct geometrical and other mathematical figures (including graphs).

Swail Dot Inverter--a device for embossing raised dots on Braille paper. This aid can be used by students to construct raised drawings.

APH Number Line Device--a device that facilitates the understanding of number lines.

Embossed Graph Sheets--manila sheets embossed with grids of either 1/2", 3/4", or 1" squares.

The following aid is available from the American Foundation for the Blind:

Sewell Drawing Kit--a kit consisting of a rubber board, a stylus, and plastic drawing sheets. This aid can be used by students to construct raised drawings.

HV1705 Barth, John L. Berla,
B282 Tangible Graph[illegible]s:
B455 Teacher's Guide.
T
c.1

DATE DUE

AMERICAN FOUNDATION FOR THE BLIND
15 WEST 16th STREET
NEW YORK, N.Y. 10011